m-ary Quasi-overlap functions on n-dimensional vectors

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Abstract. Information aggregation process is a fundamental procedure when combining or aggregating different information structures into a single one, through of an aggregation function that performs this task. In this perspective, Paiva et al. [2] introduced the notions of Quasi-overlap functions on bounded lattices as a particular instance of aggregation functions that express the overlapping degree between two lattice values and in which the continuity condition is not required.

Until now different approaches that extend the concept of the Quasi-overlap functions have been introduced for aggregating different structures, such as on finite chains setting [3], bounded posets [4] and set-based extended settings [5].

However, there are situations in wich the data considered for aggregation are vectors, which store n-dimensional information. An example of an application where vector information is used are Long Short-Term Memories (LSTM) [6] which are a type of recurrent neural networks and a powerful tool for modeling sequential data, such as time series [7, 8] and natural data language [9, 10]. In this context, in [11] the authors proposed the Vector Choquet Integral (VCI): an n-dimensional extension of the discrete Integral shock-type, such that the inputs are n-dimensional vectors and retrieving an n-dimensional vector as output, introducing in this way a VCI-LSTM architecture to deal with two problems: sequential image classification and text classification.

Inspired by this, in this work, we generalize the notion of Quasi-overlap functions to admit an input of arity m composed by n-dimensional vectors, which produces an n-dimensional vector output. Moreover, we study

some vital properties and construction methods of n-dimensional vector m-ary quasi-overlap functions.

Keywords: Information aggregation process · Quasi-overlap functions · n-dimensional vectors

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