Fuzzy Modality: From Algebraic Interpretations to Quantum Simulations via Qiskit Platform

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***Abstract*— This work provides an interpretation of the necessity and possibility fuzzy modal operators based on quantum computing. Moreover, we model these connectives on a quantum computing environment, using quantum registers and *qubits* in the IBM Qiskit software. The simulations enable a better comprehension of the evolution of quantum circuits modeling fuzzy sets generated by modal operators.**

I. INTRODUCTION

Fuzzy Logic (FL) is a powerful and handful tool, mathematically modeling the vagueness and uncertainty of information inherent to human thinking through the Fuzzy Set Theory (FST) with huge technological applications. The FL representability is greater than the classic logic (restricted to the binary set 0,1) since each element of a fuzzy set may belong to all sets, with an associated membership degree (MD) leveraging by the entire unit interval [0,1].

Additionally, Quantum Computing (QC) provides an exponential advantage to processing and storing fuzzy data by mapping MD related to single attributes to *qubit* states and multi-attribute objects as tensor products of *qubit* states. Thus, the quantum simulation of fuzzy systems seems attractive for future research, by taking the fuzzy complement, intersection, and union operations, respectively, modeled by the notions of fuzzy negations, triangular norms, and triangular conorms, on the related MD. Also, their quantum interpretation is formalized by performing projective measures over multivalued quantum transformations, mainly considering Controlled Not and Toffoli gates. By demanding the fuzzy connectives' interpretation to potentialize the measure operations' action, we apply the quantum superposition and entangled states in the MD interpretation of fuzzy data. The interpretation of the modal-like necessity and possibility fuzzy operators, as effective operators in modeling and reasoning, involves contexts of uncertainty. These modal operators may be used in various applications, from classical to multivalued logic, interpreting the truth-stressing or truth-depressing hedges depending on whether they fortify or soften a proposition's meaning. Then, based on different approaches for the necessity and possibility operators, we interpret fuzzy modal operators based on quantum computing, which are the quantum interpretation of ρ-possibility and η-necessity fuzzy modal connectives. Moreover, we apply this theoretical approach in case studies addressing the simulation of quantum circuits modeling such fuzzy algorithms on a quantum computing environment, using quantum registers and qubits in the IBM Qiskit software. The simulations enable a better comprehension of the evolution of quantum circuits modeling fuzzy modal operators.

The paper is organized as follows. Section II brings the main concepts used in the paper, which are relevant to its comprehension. Fuzzy modal operators are introduced in Section III, and in Sect. IV, their modeling in QC is provided. The quantum simulation of the fuzzy modal connectives is discussed in Sect. V, including an application by agents' interactivity. Finally, the last section outlines our concluding remarks and future works.

II. PRELIMINARIES

To highlight the synergy between QC and FST, the main concepts are recalled in the following subsections.

A. Quantum Computing

In QC, the *qubit* is the basic information unit, being the simplest quantum system, defined by a unitary and bi-dimensional state vector. *Qubits* are generally described, in Dirac's notation [9], by the expression: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where the coefficients α and β are complex numbers for the amplitudes of the corresponding states in the computational basis (state space), respecting the condition $|\alpha|^2 + |\beta|^2 = 1$, which guarantees the unitary of the quantum system state vectors, represented by $(\alpha, \beta)^t$ [7].

The quantum system state space with multiple *qubits* is obtained by the tensor product of the space states of its subsystems. Considering a quantum system with two *qubits*, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and $|\varphi\rangle = \gamma|0\rangle + \delta|1\rangle$, the state space comprehends the tensor product given by $|S\rangle = |\psi\rangle \otimes |\varphi\rangle = \alpha \cdot \gamma|00\rangle + \alpha \cdot \delta|01\rangle + \beta \cdot \gamma|10\rangle + \beta \cdot \delta|11\rangle$. A quantum system state transition is performed by controlled and unitary transformations associated with orthogonal matrices of 2^N-order, with N being the number of *qubits* within the system, preserving norms, and thus, probability amplitudes [6]. Similarly, a Quantum Transformation (QT) of multiple *qubits* can

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be obtained by the tensor product performed over unitary QT. For instance, the *NOT* operator (*Pauli-X* QT) and its application over 1-dimensional quantum system is given as:

$$\mathrm{X}|\psi\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}.$$

The tensor product defined by the Hadamard QT ($H \otimes H$) generates a superposition mathematically described by:

$$H \otimes H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

And, the action of a Toffoli QT, described as a controlled operation for a 3-dimensional quantum system is given by:

$$\mathrm{T}|S_0\rangle = \begin{pmatrix} \mathrm{Id} & 0 \\ 0 & \mathrm{X} \end{pmatrix}(\psi \otimes \varphi \otimes \sigma) = \psi \otimes \varphi \otimes X(\sigma).$$

So, it applies the *NOT* operator to the third *qubit* $|\sigma\rangle$ if the current states of the first two *qubits* are both $|1\rangle$.

The information from a quantum system is provided by measurement operators, defined by a set of linear operators (M_m) called projections. The index m refers to the possible measurement results. If the state of a 1-dimensional quantum system is $|\psi\rangle$ immediately before the measurement, the probability of an outcome occurrence is given by

$$p(|\psi\rangle) = \frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^{\dagger}M_m|\psi\rangle}}.$$

Measuring a *qubit* $|\psi\rangle$ with $\alpha, \beta \neq 0$, the related probability of observing $|0\rangle$ and $|1\rangle$ are given by:

$$p(0) = \langle\phi|M_0^{\dagger}M_0|\phi\rangle = \langle\phi|M_0|\phi\rangle = |\alpha|^2;$$
$$p(1) = \langle\phi|M_1^{\dagger}M_1|\phi\rangle = \langle\phi|M_1|\phi\rangle = |\beta|^2.$$

After the measuring process, the quantum state $|\psi\rangle$ has $|\alpha|^2$ probability of being in state $|0\rangle$ and $|\beta|^2$ probability of being in state $|1\rangle$. And, in multidimensional systems, the operators M_m^n and $p_N(m)$ denote the m-projection and the corresponding probability measure, over the n-*qubit*.

B. Fuzzy Set Theory

A fuzzy set $\mathcal{A}$ concerning the universe of discourse $\chi \neq \emptyset$ is given as: $\mathcal{A} = \{(\mathrm{x}, f_{\mathcal{A}}(\mathrm{x}))\colon \mathrm{x} \in \chi\}$, considering $f_{\mathcal{A}}\colon \chi \to [0,1]$ as its membership function [11]. In addition, the complement of a fuzzy set $\mathcal{A}$, defined as $\mathcal{A}' = \{(\mathrm{x}, f_{\mathcal{A}'}(\mathrm{x}))\colon \mathrm{x} \in \chi\}$, has the membership function $f_{\mathcal{A}'}\colon \chi \to [0,1]$, $f_{\mathcal{A}'}(\mathrm{x}) = 1 - f_{\mathcal{A}}(\mathrm{x})$, $\forall \mathrm{x} \in \chi$.

The fuzzy set can be defined by fuzzy connectives. Here, we study fuzzy negations together with modal operators.

Definition 1: Let $N\colon [0,1] \to [0,1]$ be a **fuzzy negation**, so N satisfies, for all $x, y \in [0,1]$, $N1$: $N(0) = 1$ and $N(1) = 0$; and $N2$: $x \leq y \Rightarrow N(x) \geq N(y)$. Additionally, a strong fuzzy negation is involutive, i.e. $N3$: $N(N(x)) = x$.

III. Fuzzy Modal Operators

In this section, we seek to obtain the fuzzy modal possibility, necessity, and impossibility connectives, discussing their algebraic expressions and main prooperties.

Definition 2: [5] Let $N_1, N_2\colon [0,1] \to [0,1]$ be involutive fuzzy negations. The functions $\rho, \eta\colon [0,1] \to [0,1]$ verifying the corresponding properties, for all $x, y \in [0,1]$:

ρ_1: $\rho(0) = 0$; η_1: $\eta(1) = 1$;
ρ_2: $x \leq y \Rightarrow \rho(x) \leq \rho(y)$; η_2: $x \leq y \Rightarrow \eta(x) \leq \eta(y)$;
ρ_3: $x \leq \rho(x)$; η_3: $\eta(x) \leq x$;
ρ_4: $N_1(\rho(N_1(x))) = \eta(x)$; η_4: $N_2(\eta(N_2(x))) = \rho(x)$.

are called fuzzy modal possibility and fuzzy modal necessity operators, respectively.

Let $A : [0,1] \to [0,1]$ be an aggregation function [2]. Additional properties can be demanded, such as:

ρ_5: $\rho(\eta(x)) = x$; η_5: $\eta(\rho(x)) = x$;
ρ_6: $\rho(\eta(x)) = \eta(x)$; η_6: $\eta(\rho(x)) = \rho(x)$;
ρ_7: $\rho(1) = 1$; η_7: $\eta(0) = 0$.
ρ_8: $A(\rho(x), \rho(y)) = \rho(A(x,y))$; η_8: $A(\eta(x), \eta(y)) = \eta(A(x,y))$.

Proposition 1: Let $N_1(x) = 1 - x^n$ and $N_2(x) = 1 - \sqrt[n]{x}$ be fuzzy negations. The functions $\eta_n, \rho_n\colon [0,1] \to [0,1]$,

$$\eta_n(x) = x^n \text{ and } \rho_n(x) = \sqrt[n]{x}, \forall n \in \mathbb{N}, \forall x \in [0,1]. \quad (1)$$

define the possibility and necessity fuzzy connectives which verify the 5th and 7th properties. And, when $A \in \{\max, \min, T_P\}$ are aggregation functions, with $T_P(x,y) = x \cdot y$ then ρ_n-modal and η_n-modal fuzzy connectives verify the distributive properties, i.e., ρ_8: and η_8, respectively.

Proof: Straightforward. ■

Examples 1: Taking $n = 2$ in Eq. (1), and the fuzzy negations $N_1(x) = 1 - x^2$ and $N_2(x) = 1 - \sqrt{x}$. So, ρ_2-modal and η_2-modal fuzzy connectives are given by $\rho_2(x) = \sqrt{x}$ and $\eta_2(x) = x^2$.

Proposition 2: Let $\rho, \eta : [0,1] \to [0,1]$, $n \in \mathbb{N}^*$. The classes of functions given by the expressions:

$$\rho_{[n]}(x) = \frac{x}{n} \text{ and } \eta_{[n]}(x) = \frac{(n-1)+x}{n}, \quad (2)$$

define the possibility and necessity operators, respectively.

Proof: Straightforward. ■

Examples 2: By Eq. (2) and taking $n = 2$, the necessity and possibility operators can be expressed as:

$$\rho_{[2]}(x) = \frac{x}{2} \quad \text{and} \quad \eta_{[2]}(x) = \frac{x+1}{2}. \quad (3)$$

Proposition 3: Let $N_1(x) = (1-x)^2$ and $N_2(x) = 1 - \sqrt{x}$. The functions $\underline{\rho}, \underline{\eta}\colon [0,1] \to [0,1]$, expressed by

$$\underline{\rho}(x) = 2x - x^2 \text{ and } \underline{\eta}(x) = 1 - \sqrt{1-x}, \quad (4)$$

define the ρ_n-possibility and η_n-necessity fuzzy modal connectives verifying the 5th and 7th properties.

IV. Quantum Modeling of Fuzzy Modal Connectives

The modeling of fuzzy sets based on quantum computing was first introduced in [8]. Henceforth, many results to model fuzzy connectives have been developed [1], [10], notably, an application on interactive of humanoids agents describing dilemmas of game theory as seen in [3] and [4].

The next propositions express the main conditions guaranteeing the quantum interpretation of the $\underline{\rho}$-possibility fuzzy operators. Analogously, such interpretation can be defined for the other ones.

Proposition 4: Let $|S_{f_A}\rangle = \sqrt{1-f_A(\mathrm{x})}|0\rangle + \sqrt{f_A(\mathrm{x})}|1\rangle$ be a quantum register related to a fuzzy set $\mathcal{A}$. The quantum representation of the possibility fuzzy operator $\rho_{[2]}$, given in Eq.(3-a) and related to $\mathcal{A}$ is given by the quantum register:

$$|S_{\rho_{[2]}}\rangle = \mathrm{M}_3^1 \circ \mathrm{T}_3^{1,2}(|S_{f_A}\rangle \otimes H|1\rangle \otimes |0\rangle) \quad (5)$$

Proof: When $|S_{f_A}\rangle = \sqrt{1-f_A(\mathrm{x})}|0\rangle + \sqrt{f_A(\mathrm{x})}|1\rangle$ is the quantum register related to a fuzzy set $\mathcal{A}$, it holds:

$$\begin{aligned}
|S_{\rho_{[2]}}\rangle &= \mathrm{M}_1^3 \circ \mathrm{T}_3^{1,2}(|S_{f_A}\rangle \otimes H(|1\rangle) \otimes |0\rangle) \\
&= \mathrm{M}_1^3 \circ \mathrm{T}_3^{1,2}((\sqrt{1-f_A(\mathrm{x})}|0\rangle + \sqrt{f_A(\mathrm{x})}|1\rangle) \otimes (|0\rangle + \frac{\sqrt{2}}{2}(-|1\rangle) \otimes |0\rangle) \\
&= \mathrm{M}_1^3 \circ \mathrm{T}_3^{1,2} \frac{\sqrt{2}}{2}\Big(- \sqrt{f_A(\mathrm{x})}|110\rangle + \sqrt{f_A(\mathrm{x})}|100\rangle - \sqrt{(1-f_A(\mathrm{x}))}|010\rangle \\
&\qquad + \sqrt{(1-f_A(\mathrm{x}))}|000\rangle\Big) \\
&= \mathrm{M}_1^3 \frac{\sqrt{2}}{2}\Big(- \sqrt{f_A(\mathrm{x})}|111\rangle + \sqrt{f_A(\mathrm{x})}|100\rangle - \sqrt{(1-f_A(\mathrm{x}))}|010\rangle \\
&\qquad + \sqrt{(1-f_A(\mathrm{x}))}|000\rangle\Big).
\end{aligned}$$

So, when $f_A(\mathrm{x}) = x$, a measure M_1^3, performed on $|1\rangle$ at the 3rd *qubit*, returns the classical state $|111\rangle$, with probability $p_1 = \frac{f_A(\mathrm{x})}{2} = \frac{x}{2}$. It provides an interpretation of the MD of an element $\mathrm{x} \in U$ in the fuzzy set $\mathcal{A}$, obtained by the necessity operator defined in Eq.(3-a). Analogously, the measure performed on $|0\rangle$ at the 3rd *qubit* returns the probability $p_0 = 1 - \frac{x}{2}$ and the final state: $\frac{1}{\sqrt{2-x}}(\sqrt{x}|100\rangle - \sqrt{(1-x)}|010\rangle + \sqrt{(1-x)}|000\rangle)$. ∎

Proposition 5: Let $|S_{f_A}\rangle = \sqrt{1-f_A(\mathrm{x})}|0\rangle + \sqrt{f_A(\mathrm{x})}|1\rangle$ be a quantum register related to a fuzzy set $\mathcal{A}$. The quantum representation of the fuzzy set defined by $\underline{\eta}$-necessity operator, given in Eq.(4-a), is given by the quantum register:

$$|S_{\underline{\rho}}\rangle = \mathrm{M}_1^3 \circ \mathrm{T}_3^{1,2} \circ \mathrm{CNot}_3^1 \circ \mathrm{CNot}_3^2(|S_{f_A}\rangle \otimes |S_{f_A}\rangle \otimes |0\rangle) \quad (6)$$

Proof: Firstly, let $|S_0\rangle = |S_{f_A}\rangle \otimes |S_{f_A}\rangle \otimes |0\rangle$. Then

$$\begin{aligned}
|S_0\rangle &= (\sqrt{1-f_A(\mathrm{x})}|0\rangle + \sqrt{f_A(\mathrm{x})}|1\rangle) \otimes (\sqrt{1-f_A(\mathrm{x})}|0\rangle + \sqrt{f_A(\mathrm{x})}|1\rangle) \otimes |0\rangle \\
&= \sqrt{(1-f_A(\mathrm{x}))^2}|000\rangle + \sqrt{(1-f_A(\mathrm{x})f_A(\mathrm{x})}|010\rangle + \sqrt{f_A(\mathrm{x})^2}|110\rangle \\
&\quad - \sqrt{f_A(\mathrm{x})(1 f_A(\mathrm{x}))}|100\rangle \\
|S_1\rangle &= \sqrt{(1-f_A(\mathrm{x}))^2}|000\rangle + \sqrt{(1-f_A(\mathrm{x})f_A(\mathrm{x})}|010\rangle + \sqrt{f_A(\mathrm{x})^2}|111\rangle \\
&\quad + \sqrt{f_A(\mathrm{x})(1-f_A(\mathrm{x}))}|101\rangle \\
|S_2\rangle &= \sqrt{(1-f_A(\mathrm{x}))^2}|000\rangle + \sqrt{(1-f_A(\mathrm{x})f_A(\mathrm{x})}|011\rangle + \sqrt{f_A(\mathrm{x})^2}|110\rangle \\
&\quad - \sqrt{f_A(\mathrm{x})(1-f_A(\mathrm{x}))}|101\rangle \\
|S_3\rangle &= \sqrt{(1-f_A(\mathrm{x}))^2}|000\rangle + \sqrt{(1-f_A(\mathrm{x})f_A(\mathrm{x})}|011\rangle + \sqrt{f_A(\mathrm{x})^2}|111\rangle \\
&\quad - \sqrt{f_A(\mathrm{x})(1-f_A(\mathrm{x}))}|101\rangle
\end{aligned}$$

So, if $f_A(\mathrm{x}) = x$, a measure M_1^3 returns:

$$\frac{1}{2x-x^2}\Big((x|111\rangle + \sqrt{x(1-x)}|011\rangle + \sqrt{(1-x)x}|101\rangle\Big) \quad (7)$$

with probability $p_1 = 2x - x^2$, interpreting the MD of an element $\mathrm{x} \in U$ in the fuzzy set $\mathcal{A}$, obtained by the necessity operator defined in Eq.(4-b). Finally, the measure performed on $|0\rangle$ in the 3rd *qubit* returns the superposition state $|111\rangle$, with probability $p_0 = 1 - 2x - x^2 = (1-x)^2$. It interprets the non-membership degree (nMD) in the fuzzy set $\mathcal{A}$, defined by the $\underline{\rho}$-necessity operator, concluding this proof. ∎

V. Quantum Simulation of Fuzzy Modal Connectives

Next, a case study depicts an implementation of a quantum-fuzzy modal interpretation, considering the Qiskit framework. Take $|S_{f_{2A}}\rangle = \frac{\sqrt{2}}{2}(|0\rangle + |1\rangle)$, interpreting $f_A(\mathrm{x}) = 0.5$ as the MD of an element x in the fuzzy set $\mathcal{A}$, generated by the action of a modal operator. In the Qiskit simulator, input *qubits* are initialized as $|0\rangle$.

Simulation based on the $\rho_{[2]}$ fuzzy modal operator

Based on results from Prop. 4, we have the states:

1. $|S_{\rho_{[2]}}\rangle_1 = |111\rangle$, and probability $p_1 = \frac{1}{4}$;
2. $|S_{\rho_{[2]}}\rangle_0 = \frac{\sqrt{3}}{3}(|100\rangle - |010\rangle + |000\rangle)$, and $p_0 = \frac{3}{4}$.

These results are compatible with the graphical descriptions in Figures 1 and 2, presenting the simulated circuit and the histogram generated by the measure operations (1,000× executions). So, $p_1 = \frac{734}{1000} \cong \frac{3}{4}$ and $p_0 = \frac{266}{1000} \cong \frac{1}{4}$.

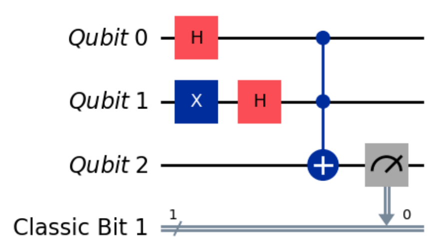

Fig. 1. $\rho_{[2]}$-Circuit.

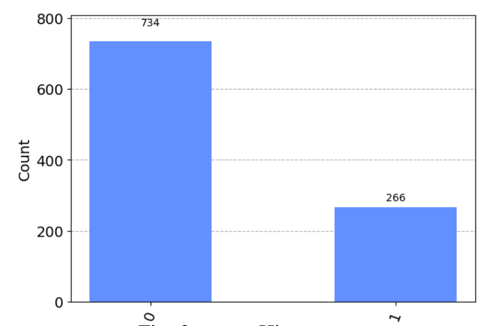

Fig. 2. $\rho_{[2]}$-Histogram.

By Prop. 5, taking $|S_{f_{2A}}\rangle$ in Eq. (7), the final states are:

1. $|S_{\underline{\rho}}\rangle_1 = \frac{2}{3}((|111\rangle + |011\rangle + |101\rangle)$, and $p_1 = \frac{3}{4} = 0.75$;
2. $|S_{\underline{\rho}}\rangle_0 = \frac{\sqrt{3}}{3}(|111\rangle + |011\rangle + |101\rangle)$, and $p_0 = 1 - \frac{3}{4} = 0.25$.

VI. Case Study: PP-P Interactivity Modeling

The PP-P problem is a social dilemma considering two police officers and one prisoner. This problem debates situations where two individuals can benefit from cooperation, having the temptation to act in their interest. So, the fuzzy sets represent the strategies related to a prisoner facing the officer's behavior, interpreted by quantum states and operators in the quantum circuit model.

In this case study, the X_1 officer humor is "possibly friendly" and the X_2 officer humor is "unfriendly". For that, let $f_{A_1}(X_1) = x_1$, $f_{A_1}(X_2) = x_2$ and $f(Y) = y$ be the MD of the officers X_1 and X_2 and prisoner Y. The fuzzy expression for the PP-P interaction, interpreted by the Xor connective, is given by $y = x_1 + x_2 - 2x_1x_2$. By the action of the fuzzy possibility operator on the agent X_1, $\rho_{[2]}(x_1) = \frac{x_1}{2}$, then we have $y = \frac{1}{2}(x_1 + 2x_2 - x_1x_2)$. In particular, when $x_1 = \frac{1}{2}$ and $x_2 = 0$, then $\rho_{[2]}(x_1) = \frac{1}{4}$ implying that $y = \frac{1}{4}$.

The input data are initialized as $|0\rangle$. Then, $F_{A_1}|0\rangle = \sqrt{1-x_1}|0\rangle + \sqrt{x_1}|1\rangle$ and $F_{A_2}|0\rangle = \sqrt{1-x_2}|0\rangle + \sqrt{x_2}|1\rangle$.

See in Fig. 3 the quantum circuit validated by Qiskit simulations, and the corresponding histogram in Fig. 4.

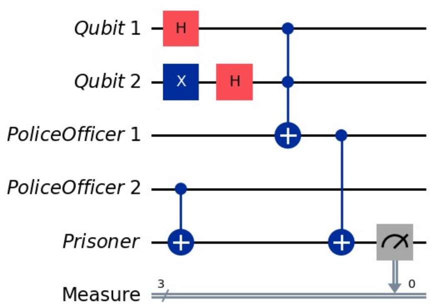

Fig. 3. η_2-Circuit in the PP-P interactivity.

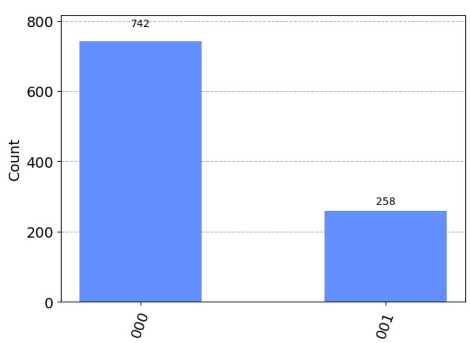

Fig. 4. η_2-Histogram in the PP-P interactivity.

In this case, the input data is modelled as $S_0 = |00000\rangle$. The resulting entangled state obtained from evolution from S_1 to S_4 is given as follows: $S_4 = \frac{1}{2}(|00000\rangle - |01000\rangle + |10000\rangle - |11101\rangle$. Thus, the measure applied to the 5$^{\text{th}}$ qubit returns state $|11101\rangle$ and $p_1 = \frac{1}{4}$. Therefore, the measure applied to the 5$^{\text{th}}$ *qubit* on $|1\rangle$ provides the interpretation at the final stage, with 25% of probability for the prisoner's cooperative behavior.

Such analysis shows a direct influence of the quantum superposition of the police officers, who are entangled with the prisoner P. This case is modeling the modality applied over the X_1 officer behavior, interpreting the "possible friendly" linguist term.

So, this interpretation can be extended for modeling the interaction between multiple police agents and prisoners.

VII. Conclusion

This study introduces new fuzzy modal operations concerning their main algebraic properties and considers fuzzy negations and aggregation functions to construct such operators. Besides, we provide the representation of fuzzy modal classes through concepts of QC related to fuzzy sets generated by such connectives, which are expressed by quantum registers and composition among QT operations.

Therefore, the presented work provides new information technologies based on the quantum-fuzzy approach, consolidating the modeling of fuzzy algorithms which can be simulated over quantum computers.

This new approach deals with inaccurate data and the imprecision inherent to rule-based systems, but it is performed according to concepts from quantum mechanics, taking advantage of the QC and modalities of FL.

Ongoing work includes further investigations on other types of modalities, focusing on the applications integrated into machine learning and artificial intelligence as the simulations of emotion intensities of humanoid robots [4].

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