Degree of ordering for intervals

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Understanding the relationships between intervals is crucial in various fields, such as mathematics and computer science. Additionally, in real-world applications such as scheduling tasks or managing resources, the order of intervals can have significant implications for decision-making processes.

Despite their importance, there is not always a clear way to measure the order between intervals, especially when considering the different types of relationships they may have [1, 2]. In this study, we explore partial orders among closed intervals on the real line and define a degree of ordering associated with these relations.

We particularly focus on two significant cases:

1) Lattice order, which is the classical order used to measure when one interval is smaller than another. The lattice order usually provides a foundational framework for comparing intervals and determining their relative positioning on the real line. Nonetheless, it is not always possible to compare intervals with it as it is a partial order;

2) Content relation: this relationship considers the elements contained within each interval and how they overlap. Unlike the lattice order, which focuses on the relative size of intervals, the content relation provides insight into the intrinsic properties of intervals and their internal structure.

We investigate how the nature of these relations determine a measure of the degree of ordering for intervals. To illustrate these concepts, we provide several examples showcasing such measures. In all cases, we have considered the epistemic interpretation of an interval to define the different families of measures.

Moreover, we can use these studies to generate measures of subsethood and embedding [3], respectively, for interval-valued fuzzy sets, by aggregating the degree of ordering associated with the two intervals describing the membership function of the two compared sets at any point of the referential.

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