A categorical duality for finite semilinear Hilbert algebras

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Abstract. Hilbert algebras are the implicative reducts of Heyting algebras, hence semilinear Hilbert algebras are the implicative reducts of Gödel algebras. In this work, we present a categorical duality both for bounded and unbounded finite semilinear Hilbert algebras (with homomorphisms), obtained by suitably adapting and enriching the well-known duality between finite Gödel algebras and finite forests with order-preserving open maps.
In particular, the main idea is to characterise elements of any finite (unbounded) semilinear Hilbert algebra H in the following way: if G is a finite Gödel hoop then G is a subdirect product of linearly ordered Gödel hoops. For any generator a in a minimal set of generators of G, an a-monochromatic element of G is an element that, in the subdirect representation, has each component equal to the top or to the corresponding component of a. The set of all monochromatic elements equipped with the Gödel implication is a Hilbert algebra and, further, the universe of any semilinear Hilbert algebra is the set of monochromatic elements of a Gödel hoop.
We shall then apply the duality to compute coproducts of finite semilinear Hilbert algebras and determine the structure of finitely generated free algebras. Interestingly enough, we obtain an explicit expression for the cardinality of the free n-generated semilinear Hibert algebra (either bounded or not). This is in sharp contrast with the cases of free Gödel algebras and hoops, where, as far as the authors know, the cardinalities are found by computing recurrence expressions.

Keywords: Hilbert algebras · Gödel algebras · Categorical duality.