An Exploration of The Weighted Quantile Approach in Probabilistic Fuzzy Inference*

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Abstract. In the paper "Significance measures for rules in probabilistic-fuzzy inference systems based on fuzzy transform, Fuzzy Sets Syst. 467 (2023)", N. Madrid proposed probabilistic-fuzzy IF-THEN rules, modelled similarly to the fuzzy IF-THEN rules used in the Takagi-Sugeno fuzzy inference system, where the consequences take the form of quantile functions. In this way, the output of each rule provides information on the distribution of output values corresponding to input values that meet the conditions specified in the antecedent. Using a linear-like combination, a suitable quantile function can be obtained for any input from a given space. This method brings a new tool to quantile regression and, among other things, can be used in time series analysis, where, for example, the moving median is a well-known technique for trend estimation. In the above-mentioned paper, the quantile functions are derived from the data using the so-called quantile fuzzy transform, which was presented in the paper "ML. Guerra et al. Quantile and expectile smoothing based on L1-norm and L2-norm fuzzy transforms, Int. Jour. of App. Reas., 107, 2019, 17-43" with an algorithm to minimize the weighted quantile formula. This means that the quantile function in the consequence of each rule is optimized separately (i.e., a best local fit), which differs from the Takagi-Sugeno approach, where the consequences of rules are optimized globally. From a computational point of view, this approach is much simpler than the Takagi-Sugeno approach because the minimization procedure uses only the weights from one fuzzy set in the IF-THEN rule under consideration.
The main focus of this contribution is the analysis of the weighted quantiles and the introduction of a very simple and fast method for their determination, in contrast to the original approach. This method is implemented in an algorithm for probabilistic-fuzzy inference systems that allows us to derive weighted quantiles for any element or interval in a

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given input domain for a fixed probability, for example, a median or median function if the probability is equal to 0.5. In addition, we can determine a quantile function for any fixed element in a given domain. Subsequently, we investigate whether the quantile functions obtained in this way from the sample data are estimations of the original quantile functions derived from given distributions. The second part aims to compare our proposed approach with existing methods, such as the weighted version of the traditional quantile estimator, the Harrell–Davis quantile estimator, or its trimmed modification. Using practical examples, we demonstrate the effectiveness of our approach and show that their resulting estimates are consistent with data trends and recover quickly from point changes. Finally, we discuss the possibility of following the Takagi-Sugeno approach with global optimization of quantile functions.

Keywords: Weighted quantiles · Quantile functions · Quantile fuzzy transform · Probabilistic-fuzzy inference · Takagi-Sugeno approach.