F-transform on triangulated domain

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The effectiveness of the F-transform [1] has been demonstrated across a spectrum of applications, including signal and image processing [2]. Despite its stable theoretical foundation, this method can be further developed. In general, the technique of F-transform is flexible enough to be adapted to many specific problems. The development is mainly associated with the choice of the main parameter - a fuzzy partition of the universe of discourse.

According to the original paper [1], the F-transform was introduced in the one-dimensional Euclidean space and later extended to the two-dimensional Euclidean space. In most of the previous applications to image processing, the method was utilized within rectangular domains due to the inherent rectangular shape characteristic of images.

In the view of the majority of applications in image processing, future developments of this method require to process domains of other types than only rectangular. In the literature, one can find various extensions of the F-transform of functional objects defined on d-dimensional domains up to domains with the structure of manifolds [3].

However, to be able to utilize the developed image processing techniques, we propose to generalize F-transform for any compact connected domains in 2D space. Therefore, we are able to create analytical model of the analysed object using inverse F-transform representation. Inspired by computer graphics with a vision to apply F-transform on 3D objects in the future, we decided to construct a fuzzy partition on a triangulated domain. In detail: the main problem considered in this contribution is to develop the F-transform methodology for functions (images) on complicated domains that allow Delaunay triangulation. That is, we will propose a way to calculate the components of the F-transform and then design the inverse formula. We will then analyse the proximity between the initial functional object and its inverse F-transform representation.

Our experiments start with circular domains with uniformly distributed nodes. We use Delaunay triangulation [4] of the domain. Vertices of the triangulation represent nodes of fuzzy partition, so each vertex of the triangulation corresponds to one basic function. Basic functions of piecewise linear type are considered so the domain is being covered by n-sided pyramids since more complex shapes should not have a significant impact on the result.

The number of nodes is tightly connected with the proximity value. In other words, the focus is to obtain the best possible proximity with the smallest number of nodes employed. We propose to estimate proximity based on an action of the Laplace operator applied to the initial functional object. Established on our

previous research [5], we evaluate the Laplace operator applied to a function as difference between this function and its F-transform components taken at partition nodes.

To confirm the method's efficiency, the obtained inverse F-transform representation was compared with the original functional object. The deviations were minor with respect to the number of fuzzy sets contained in the fuzzy partition. In conclusion, we describe types of functions where this method works efficiently.

Keywords: F-transform · fuzzy partition · Laplace operator · triangulation

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