Sup-T composition of fuzzy implications from different families

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Fuzzy implications extend the classical implication from $\{0,1\}$ to the multi-valued setting. They are defined as binary functions $I\colon [0,1]^2 \to [0,1]$ that decrease in the first variable, increase in the second variable and satisfy the boundary conditions $I(0,0) = 1, I(1,1) = 1$ and $I(1,0) = 0$.

Fuzzy implications, along with other fuzzy logic connectives, play a significant role both in the theory and applications of fuzzy logic connectives. Among the various applications, the following composition of fuzzy implications is used simultaneously both in theory and applications [1, 2]: Let I, J be two fuzzy implications and $*$ be a t-norm. Then sup–$*$ composition of I, J is given as follows:

$$(I \overset{*}{\circ} J)(x,y) = \sup_{t\in[0,1]} (I(x,t) * J(t,y)), \qquad x,y \in [0,1]\ . \tag{COMP}$$

Note that $I \overset{*}{\circ} J$ is a fuzzy implication if and only if $(I \overset{*}{\circ} J)(1,0) = 0$ (see Theorem 6.4.4 in [2]).

Since the sup–$*$ composition of fuzzy implications does not always give a fuzzy implication and due to its applicational demand, it is always essential to investigate the triples $(I, J, *)$ for which the composition is a fuzzy implication.

In this work, we investigate the necessary and sufficient conditions on fuzzy implications such that their sup–$*$ composition becomes a fuzzy implication. Towards this, we restrict the investigations to some well established families of fuzzy implications. Further, we explore the subsemigroups of these famlies of fuzzy implications.

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