Averaging functions and admissible orders applied to the Interval Shortest Path Problem

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The Shortest Path Problem (SPP) has been and continues to be a primary concern in the field of optimization. With the most basic formulation, multiple situations are modeled, and there are numerous variations that adapt the problem to diverse contexts. Optimization models for all these situations seem to provide good quality and practical solutions. However, most models tend to overlook the uncertainty component that is always present in real life. The considered uncertainty in the present work is reflected in the costs of the arcs that are in the network and may arise from data collection itself or external factors affecting the problem that cannot be controlled or predicted such as traffic or weather conditions.

In recent studies, data imprecision is represented using fuzzy theory, giving rise to the Fuzzy Shortest Path Problem (FSPP) (see for example, [1]). Nevertheless, part of the effort in the present work has been focused on facilitating data interpretation for non-experts. To achieve this, uncertainty has been expressed in the form of intervals. In this case, the cost of traversing an arc in a graph is given as an interval, meaning that the actual cost lies between the limits of that interval. The problem, therefore, is to find the path from a source node to a target node that minimizes the sum of the intervals representing the arc costs in that path. This is denoted as the Interval Shortest Path Problem (ISPP).

The main challenge when dealing with intervals, is the absence of a general order, since there exist pairs of intervals that are not be comparable by the usual partial order in the two-dimensional euclidean space. This work studies the properties of admissible orders introduced by Bustince et al. in [2], as well as averaging functions, to address that issue, and properly incorporates them into a variation of the Dijkstra algorithm to find the minimum path between two nodes in a graph. Both techniques, admissible orders and averaging functions, are analyzed within the context of the ISPP, allowing for simplifications and giving meaning to the parameters involved. A comparison is carried out in a case study to showcase the proposed methodology and highlight the differences.

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