Embedding measures for IVFSs as an interval

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Fuzzy sets, that were widely used to model imprecision and human reasoning, force the degree of membership of every point to be a precise value. This could be a very restrictive condition in some specific cases. This criticism led some authors to define interval-valued fuzzy sets (IVFSs). They do not force the decision maker to provide a precise value to express the membership degree but allow the judge to provide a range of possible values. From the epistemic point of view [1], the aforementioned range is only a rough approximation to the actual membership degree that is a unique value but that cannot be determined in an accurate way.

Interval-valued fuzzy sets can model human behavior in a more appropriate way than fuzzy sets. However, they are also more complex structures. In order to handle these structures, new concepts have to be defined and explored. One of these new concepts is the definition of embedding. An IVFS is embedded in another one when for every element, its range of possible membership degrees is a subset of the range of possible membership degrees of the second one, this is, when the intervals that represent the possible membership degrees of the elements to the first set are included (as sets) in the intervals that represent the membership degrees of the elements to the second set. What happens when the content does not hold for some elements or given an element, not for the whole interval? It is clear that a definition that allows levels of embedding is necessary.

In a previous approach we summarized the measure of embedding of an IVFS into another one with just one value [2]. In this contribution, we propose a new definition of embedding degree where the outcome is not forced to be a number, but an interval. The underlying idea is that if we allow the decision maker to provide a range of possible belongness degrees, we should also allow the embedding degree to be a range of values. This new viewpoint forces to rethink the axioms included in the definition and to adapt them properly to the new context. After providing the new definition, we study their properties and we connect this new idea with the definition of embedding of intervals.

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