

Some solutions of the functional equation

$$A(T(x, y), S(x, y)) = A(x, y)$$

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A binary aggregation on $[0, 1]$ is a function $A : [0, 1]^2 \rightarrow [0, 1]$ which is increasing in both variables and satisfying the boundary conditions $A(0, 0) = 0$ and $A(1, 1) = 1$. Among the classes of aggregation functions, t-norms and t-conorms are two important ones, which play significant role in Aggregation theory and Fuzzy logic. For more details about the aggregations and t-norms, please refer to [1, 2].

The study of functional equations involving fuzzy logic connectives has found immense utility in advancing theory and applications. Among such functional equations, an open problem was proposed as follows in [3] as Problem 8.

Problem 1. Given a binary aggregation operator $A : [0, 1]^2 \rightarrow [0, 1]$, characterize all pairs (T, S) of a t-norm T and a t-conorm S such that for all $(x, y) \in [0, 1]^2$,

$$A(S(x, y), T(x, y)) = A(x, y), \quad (1)$$

holds.

In this work, we take up the task of investigating the solutions of Problem 1. Towards this, we show that for any choice of A , there exists at least one pair (T, S) of t-norms and t-conorms that satisfy (1) and also show that some conjugacy transformations preserve the solutions of (1). Further, we present some necessary and sufficient conditions on the operators A, T and S for the satisfaction of the functional equation (1). We present our nascent explorations on the same with suitable examples.

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