

Algebras on Continuous Archimedean T-norms

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Triangular norms (t-norms, for short) are a generalization of classical conjunction from the two valued logic to the multi-valued setting. A map $T: [0, 1] \rightarrow [0, 1]$ is called a t-norm if it commutative, associative, increasing in both the variables and has 1 as neutral element. T-norms are an important class of operators both in Aggregation theory and Fuzzy logic. For more details about the theoretical and applicational aspects of t-norms, please refer to [1].

It is well known that the usual point-wise ordering over the set of t-norms makes it a poset but not a lattice, i.e., the point-wise maximum or minimum of two t-norms need not always be a t-norm again. Recently [2] proposed a way of obtaining a lattice structure on the set of continuous t-norms. This gave both a way of generating continuous t-norms from continuous t-norms and also obtain an order on them.

Further, continuing along these lines, in this work, we define operations on the set of continuous Archimedean t-norms, which confers an algebraic structure to it, in fact, we show that we can obtain a convex cone structure. Further, we see that under a suitable operation, the set of strict t-norms (nilpotent t-norms) forms a group.

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References

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