On Approximation of Lattice-valued Functions Using Lattice Integral Transforms*

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Abstract. Lattice integral transforms for lattice-valued functions were introduced to generalize and then extend the approximation capabilities of lattice fuzzy transforms proposed by Perfilieva in her pioneering paper on fuzzy transforms, which can be used in signal and image processing, compression, denoising, data analysis, etc. Without going into details, the lattice integral transform is defined similarly to standard integral transforms such as Fourier or Hilbert transforms for a real or complex function, only the Lebesgue integral is replaced by the Sugeno-like integral and the kernel is given as a fuzzy relation. Moreover, the operation between the kernel and the transformed function takes two forms, namely multiplication and residuum in a given residuated lattice. It can be shown that lattice fuzzy transforms are special cases of lattice integral transforms where the smallest and largest fuzzy measures are considered. Perfilieva showed that the compositions of lattice fuzzy transforms are upper and lower approximations of the original functions. The same is not true for lattice integral transforms and an interesting and challenging question arises as to how we can describe the approximation of the composition of lattice integral transforms and how we can measure its quality, since we are working in the complete residuated lattice and therefore cannot use the standard apparatus for real and complex-valued functions.

In the presentation, we provide answers to the above questions. We introduce the concept of the inverse integral kernel (so-called Q-inverse for a fuzzy relation Q on the domain X of functions that are transformed) which plays a crucial role in the reconstruction of the original function using the composition of lattice integral transforms. Using Q-inverse, we generalize Perfilieva's statement on the upper and lower approximations. Next, we define the modulus of continuity in terms of similarities for the lattice-valued functions, which is the counterpart of the standard continuity modulus in the framework of complete residuated lattices. We use

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the modulus of continuity to estimate the quality of the approximation of the reconstructed function. In addition, we show that under certain conditions, the quality of approximation is greater than the Sugeno-like integral of the square of the fuzzy relation Q, so the estimation is independent on smoothness of original function measure by the modulus of continuity. As a corollary, we get that again under certain conditions on integral kernels, the extensional functions with respect to Q can be ideally reconstructed, i.e., the original and reconstructed functions coincide.

Keywords: Sugeno-like integral, integral transforms, lattice-valued functions, approximation, signal processing