

# Do Implicational Weighted Fuzzy Rules Serve Well Only for Functional Dependency?

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**Abstract.** In this study, we investigate the applicability of implicational weighted fuzzy rules beyond functional dependency.

**Keywords:** Fuzzy rules · Automated learning · Knowledge representation.

In [3], a novel model for fuzzy rules with weights was introduced. There, the weighted fuzzy rules model for fuzzy sets  $A_i$  and  $B_i$  in  $X$  and  $Y$ , respectively,  $i \in I$  ( $I$  is a finite set of indices) took the following form:

$$\text{GRules}_q^{\mathcal{D}}(x, y) = \bigwedge_{i \in I} (q(A_i, B_i) \rightarrow [A_i(x) \rightarrow B_i(y)]),$$

for all  $x \in X, y \in Y$ , where  $\rightarrow$  is a fuzzy implication,  $q$  is an implicational quantifier, i.e., a measure of dependency and causality between (fuzzy) sets based on observations from the data matrix. For the definition of  $q$  and more details on implicational quantifiers, we refer to [2, 4, 3].

As it was proved in [1], the implicational models are suitable for functional fuzzy relations. In other cases, it does not work well because joining two contradicting fuzzy rules, i.e., rules with identical antecedents  $A$  and consequents  $B_1, B_2$  such that  $B_1(y) \& B_2(y) = 0$ , for all  $y \in Y$ , leads to value 0 everywhere on the (fuzzy) Cartesian product of  $A$  and  $Y$ . Since the implicational quantifiers compare the numbers of confirming observations with rejecting ones while computing the degree of causality, consequently, the same problem applies also for the above model.

In this contribution, we propose a solution to this problem by considering quantifiers that would not take into account rejecting observations. Moreover, we provide some basic properties of the modified model.

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