F-transform Utility in the Operational-Matrix Approach to the Nonlinear Volterra Integral Equation

Tam Pham and Irina Perfilieva[0000−0003−1531−1111]

IRAFM, University of Ostrava, 30. dubna 22, 701 03 Ostrava, Czech Republic
{thi.pham,irina.perfilieva}@osu.cz
http://www.osu.eu/

Nonlinear Volterra integral equations involve both nonlinear functions and integrals. The general form of a non-linear Volterra equation of the second kind has the form

$$y(t) = f(t) + \int_0^t K(s,t)\phi(y(s))ds, \tag{1}$$

where the used parameters are as follows: source $f \in C^1[0,T]$, kernel $K \in C^2([0,T] \times [0,T])$, $\phi \in C^1[\mathbb{R}]$ are known and function y is unknown. The equations of this type are used in various fields such as physics, biology, economics, and engineering to model complex systems that involve memory effects or interactions between different components. This research is a smart combination of the two methods: the F-transform and Fixed Point Theorem to approach a numerical solution of the nonlinear Volterra integral equation [2].

We propose to compute an approximate solution to the equation (1) using the F-transform [3–7]. In details, we construct an operational matrix for the Volterra integral equation as a Hadamard product of two matrices, one of which refers to the Volterra operator, and the other to its kernel. By this, the entire equation can be reduced to a simpler form. This makes our method efficient and low computational. The supporting statements, including the convergence of the method, and estimate its computational complexity will be provided.

Additionally, in order to guarantee that the nonlinear equation (1) has a unique solution, we show the applicability of the Fixed Point Theorem, which plays a key role in solving many problems in applied mathematics such as neutron transport, population biology, economics, applied mechanics, etc, see [8–10].

In parallel, our methodology is validated through the development of software that confirms the efficacy of our method on a range of examples. Our work demonstrates the joint progress of theoretical results and the development of practical software for solving complex mathematical problems.

Keywords: Volterra integral equation · Nonlinear · Fuzzy transform · Fixed Point Theorem

References

1. Perfilieva, I., Ziari, S., Nuraei R., Pham T.M.T.: F-transform utility in the operational-matrix approach to the Volterra integral equation. Fuzzy Sets and Systems **475**, (2024). https://doi.org/10.1016/j.fss.2023.108764
2. Wazwaz, A-M.: Volterra Integral Equations. Linear and Nonlinear Integral Equations, 418–430 (2022)
3. Perfilieva, I.: Fuzzy transforms: theory and applications. Fuzzy Sets and Systems **157**, (2006)

4. Perfilieva, I., Dankova, M., Bede, B.: Towards a higher degree F-transform. Fuzzy Sets and Systems **180**, (2011)
5. Perfiljeva, I., Pham, T.M.T.: CI Approach to Numerical Methods for Solving Fuzzy Integral Equations. In: Information Processing and Management of Uncertainty in Knowledge-Based Systems, (2022), pp. 282–293.
6. T. M. T. Pham, T.M.T., Janeček, J., Perfiljeva, I.: Fuzzy Transform on 1-D Manifolds. In: Biomedical and Other Applications of Soft Computing, (2022), pp. 13-24.
7. Perfilieva, I.: Fuzzy transform: Application to reef growth problem. Fuzzy Logic in Geology, 275–300 (2003)
8. Nazam, M., Arshad, M., Park, C., Mahmood H.: On a fixed point theorem with application to functional equations. Open Mathematics, 1724–1736 (2019)
9. Sharma, P.K., Sharma, S., Kaushik, J., Goyal P.: Applications of Fuzzy Set and Fixed Point Theory in Dynamical Systems. Qualitative and Computational Aspects of Dynamical Systems, (2023).
10. Guran, L., Mitrovi, Z.D., Reddy, G., Belhenniche, A., Radenovi, S.: Fixed Point Theory and Its Applications in Nonlinear Analysis and Optimization. Mathematics, (2023). https://doi.org/ISSN 2227-7390