# Pseudo-additive generators for pseudo-overlap and pseudo-grouping functions*

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Overlap and grouping functions are commutative continuous aggregation functions, defined on the unit square, and satisfying some given boundary conditions. The overlap functions, originally bivariate, were introduced in [1], where they were shown to be helpful in image processing, specifically in object recognition problems, where the best classification with respect to the background is the one with less overlap between the class object and the class background.

In [6], the symmetry (or commutative) axiom was dropped from the overlap and grouping functions by introducing the concepts of pseudo-overlap and pseudo-grouping functions. The motivation for introducing these new functions is both theoretical and applied. On the one hand, in the case of multi-criteria decision problems, symmetric functions have their limitations in modelling criteria of unequal importance. In the case of mathematical morphology, the use of non-commutative operators has a wider applicability. On the other hand, in the case of time-dependent data aggregation, the use of symmetric functions may break the natural relationship of the data.

There are various construction methods for overlap functions, and a very important way of constructing them is by means of additive generators [2]. Their relevance lies in the fact that they construct two-place aggregation functions by means of one-place functions, thus reducing the computational complexity [3, 5]. This type of generators, together with multiplicative generators, form a core of study in the field of aggregation functions, due to the simplification in the study of properties such as monotonicity, continuity and convexity [4].

Given the applicability that pseudo-overlap and pseudo-grouping functions can have, in this work we present their additive generators, called pseudo-additive generators. For this purpose, we first define the pseudo-additive oper-

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ation, present results for the generation of pseudo-overlap and pseudo-grouping functions and study under which cases the generated functions are of this family.

## References

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