General framework for intuitionistic values

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Abstract. In this contribution, we introduce a general framework of the intuitionistic values, which are sometimes referred to as the intuitionistic membership grades. This framework includes, but is not limited to, for example, the classical intuitionistic values introduced by Atanassov in his famous paper [1]; the Pythagorean intuitionistic values introduced by Yager and Abbasov [2]; the Fermatean intuitionistic values introduced by Senapati and Yager [3]; or the framework of the q-rung intuitionistic values.

Every strong negation n, i.e., a unary function $n\colon [0,1] \to [0,1]$ that is strictly decreasing, continuous and involutive, can be generated by an automorphism φ, i.e., a strictly increasing bijective mapping $\varphi\colon [0,1] \to [0,1]$, by $n(x) = \varphi^{-1}(1 - \varphi(x))$ for all $x \in [0,1]$, where φ^{-1} denotes the inverse map of φ. The class of all ϕ-intuitionistic values consists of pairs $(u,v) \in [0,1]^2$ such that $\varphi(u) + \varphi(v) \leq 1$. Equivalently, one can require $u \leq n(v)$, where n is the strong negation generated by φ. Note that, if φ is a generator of the standard negation $n(x) = 1 - x$ proposed by Zadeh in [4], given by $\varphi(t) = t$, the classical intuitionistic values are reconstructed; if $\varphi(t) = t^2$, the Pythagorean intuitionistic values are obtained, or, in general, if $\varphi(t) = t^q$, which is a generator of the Yager class of negations characterized by $n_q(x) = (1 - x^q)^(1/q)$, q-rung intuitionistic values are recovered. As a non-trivial example, one can consider the automorphism $\varphi(x) = \log_2(x+1)$, which generates the strong negation n given by formula $n(x) = (1-x)/(1+x)$ and it characterizes the corresponding type of φ-intuitionistic values which satisfy the inequality $(1+x)(1+y) \leq 2$.

A ϕ-intuitionistic fuzzy set A is a mapping assigning a ϕ-intuitionistic value to every element of the universe of discourse. If (u,v) is the intuitionistic value assigned to the element x, the value of u is referred to as the degree of membership and the value of v is referred to as the degree of non-membership of the element x to the set A. It is usual to define set operations on these intuitionistic fuzzy sets. It can be naturally observed that ϕ-intuitionistic values are in a one-to-one correspondence with the class of all closed intervals included in the unit interval $[0,1]$. It is convenient to define operations on these closed sub-intervals using a generator of a t-(co)norm and thus allowing us to define operations on the class of ϕ-intuitionistic fuzzy sets. The ϕ-intuitionistic fuzzy values are exemplified and the properties of operations on them are examined.

Keywords: Inutitionistic fuzzy sets, strong negations, triangular norms.

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