Pi-ordinal sums of transformations of copulas

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Abstract. Families of copulas that include the three basic copulas, i.e., the Fréchet-Hoeffding bounds and the independence copula (possibly as limit cases), are called comprehensive. Two possible one-parameter comprehensive families of copulas that were mentioned in [2] were related to (θ)- and $[\theta]$-transformations of copulas given by

$$C_{(\theta)}(x,y) = C(x,y) + \theta C(x,y)(C(x,y) - x - y + 1)$$

and

$$C_{[\theta]}(x,y) = C(x,y) + \theta(x - C(x,y))(y - C(x,y)).$$

Note that, in general, the result of these transformations is not necessarily a copula for all values of the parameter θ. Moreover, these transformations may lead to values outside the unit interval $[0,1]$ and thus the truncation by the Fréchet-Hoeffding bounds is needed to extend the set of the parameters, i.e., we consider transformations

$$\overline{C_{(\theta)}}(x,y) = (W(x,y) \vee C_{(\theta)}(x,y)) \wedge M(x,y) \quad \text{and}$$
$$\overline{C_{[\theta]}}(x,y) = (W(x,y) \vee C_{[\theta]}(x,y)) \wedge M(x,y).$$

For the parameter sets of the basic three copulas for the mentioned transformations we refer an interested reader to the paper [2]. In this contribution, we examine the Π-ordinal sum construction (see, e.g., [1]) under these transformations. We will examine for which parameters θ a copula is obtained and we will fully characterize the set of these parameters for those Π-ordinal sums constructed using the three basic copulas. We will exemplify these constructions and we will remark on their dependence coefficients.

Keywords: copula construction, truncation, copula transformation

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