Discrete extended aggregation functions based on extreme values

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Abstract. Aggregation functions on the string C such that the output value depends only on a small set of the largest and/or smallest values of the given inputs are introduced and investigated. The present contribution is concerned with the study of symmetric associative functions, including extended forms of t-conorm, t-norm, uninorm and nullnorm.

Keywords: aggregation function, nullnorm, t-norm, t-conorm, uninorm

After a brief introduction to discrete extended aggregation functions acting on a fixed finite chain C of numerical values or linguistic terms (e.g., *Extremely Bad, Very Bad, Bad, Fair, Good, Very Good, Extremely Good*), represented by the set $\{a_0, a_1, \dots, a_d\}$ equipped with a linear order $\prec$, so that $a_0 \prec a_1 \prec a_2 \prec \dots \prec a_d$. We focus on those based on only a few extreme values.

Consider a pair of non-negative integers (u, v), $u + v > 0$. An extended discrete aggregation function A is called a (u, v) - aggregation function if, for any $n \geq u + v$, and $\mathbf{x} \in C^n$,

$$A(\mathbf{x}) = A(x_{(1)}, \dots, x_{(u)}, x_{(n-v+1)}, \dots, x_{(n)}),$$

where (.) is a permutation such that $x_{(1)} \preceq \dots \preceq x_{(n)}$, i.e., for the evaluation of $A(\mathbf{x})$ only u smallest and v largest values of the n-tuple $\mathbf{x}$ are considered.

As a typical example, to evaluate the max operator, only one largest input value should be known, i.e., max is a (0,1) - aggregation function. Similarly, min is a (1,0) - aggregation function, and any extended idempotent uninorm U on C is a (1,1) - aggregation function. After presenting some general construction methods, we study in detail the most important binary associative functions with a neutral element e and their connection to (u, v) - aggregation functions. As already mentioned, the smallest t-conorm max $= S_M$ is (in its extended form) just a (0,1) - aggregation function, and it is a unique discrete t-conorm with this property. The drastic sum S_W is a (0,2) - aggregation function. We show that every extended t-conorm S is a (0,v) - aggregation function, where $v \in \{1, \dots, d\}$, and that the unique t-conorm that is a (0,d) - aggregation function is the Lukasiewicz t-conorm S_L. As an important consequence, we can introduce a classification of discrete t-conorms with exactly d-classes based on the associated parameter v. We also introduce some construction methods for $(0,v)$ - t-conorms. By duality, results for discrete t-conorms could be rewritten for discrete t-norms, possibly with some modifications. As a typical example of $(u,0)$ - t-norm on C, we can consider a smooth t-norm T determined by a non-trivial idempotent elements $\{a_u, a_{u+1}, \ldots, a_{d-1}\}$.

The results obtained for discrete t-conorms and discrete t-norms on C, considering the influence of extreme values, could be applied to the discrete uninorms. Recall that each extended discrete uninorm U with a neutral element $e \in \{a_1, \ldots, a_{d-1}\}$ is characterised by the underlying extended discrete t-norm T (acting on the chain $\{a_0, a_1, \ldots, a_k\}$, where $a_k = e$), the underlying discrete t-conorm S (acting on the chain $\{a_k, \ldots, a_d\}$) and the binary form of U. Then U is a (u, v) - aggregation function if and only if T is a $(u,0)$ - aggregation function and S is a $(0,v)$ - aggregation function. A similar result can be shown for nullnorms.

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