A hybrid model with K-means and ELECTRE-III to analyze countries considering prosperity indicators*

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Abstract. This article employs an original hybrid approach to evaluate countries, combining the K-means clustering algorithm with the ELECTRE-III multicriteria ranking method and Monte Carlo simulation. The aim is to rank representative alternatives constructed by the obtained clusters' centroids and taking into account the results sensibility to uncertainties related to the parameters of the modelling. Therefore, besides grouping the alternatives into homogeneous categories, which not necessarily are ordered in terms of preference, our approach ranks these clusters. The approach is applied to a dataset with 12 indicators regarding a prosperity evaluation, namely the Prosperity Index from the Legatum Institute. The results include cluster visualizations, the preference relations defined by ELECTRE-III, and the resulting ranking. Furthermore, a subsequent analysis is presented using 10,000 simulations that consider variations in the ELECTRE-III parameters and the utilization of probability distributions in order to account for uncertainty. The results demonstrate consistency with expectations, and the robustness of the rankings is confirmed by the statistics obtained from the simulations.

Keywords: Decision · ELECTRE · K-means · Monte Carlo · multicriteria · Prosperity Index

1 Introduction

The analysis of country indicators often involves several conflicting factors, such as economic growth versus environmental sustainability, social equity versus market competitiveness, and short-term gains versus long-term stability. Nethertheles, the importance of considering multiple factors other than those economic-related is increasingly clear, which may include topics such as sustainability [5,

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27], socially responsible investments [3, 9], and life quality aspects [4]. In this scenario, using mathematical models that combine multiple dimensions becomes interesting for decision-makers and analysts. By using these techniques, policy-makers can analyze a country's situation and compare it to neighboring countries or even countries with a similar economic situation. In this context, Machine Learning (ML) [13] and Multiple Criteria Decision Making/Analysis (MCDM/A) [10] techniques stand out.

ML algorithms can be classified according to how the computer learns to perform a specific task from data. For example, in Supervised Learning, the computer learns to label objects based on a set of previously labeled examples (training set). In the case of Unsupervised Learning, the goal is to discover and understand patterns in a dataset without initial labels [20, 13]. Tasks commonly solved with Unsupervised Learning techniques include clustering, descriptive statistics, and dimensionality reduction [13, 21]. K-means is one of the most known ML techniques used for clustering. This method organized the data examples (instances/objects) into clusters according to their similar distances to the clusters' centers.

In turn, MCDM/A methods provide a systematic approach to consider evaluations of alternatives based on multiple criteria and the decision maker's preferences in a decision problem [10]. The types of problems addressed (referred to in the community as "problematic") include description, choice, ordering, and classification [23]. The use of this approach has proven useful for country evaluation, as observed for instance in studies by [2, 25, 16].

Perceiving the complementary potential between ML clustering algorithms and MCDM/A ranking methods, a recent work [26] proposed the use of K-means in conjunction with the well-known ELECTRE-III method [22]. In this article, we implement this approach by adding a step to deal with uncertainties in the parameters of ELECTRE III modelling. Hence, the proposal was applied to analyze countries based on indicators of the Sustainable Development Index by the Legatum Institute, with a robustness analysis of the ranking obtained by ELECTRE-III through Monte Carlo simulations with parameter variations of ELECTRE-III obtained from probability distributions.

The remainder of this work is organized as follows. Section 2 provides a brief theoretical background with details of the K-means and ELECTRE-III methods. Section 3 presents the application of the hybrid approach to analyze countries based on sustainable development indicators. The results are discussed in Section 4. Finally, Section 5 presents the study's conclusions and prospects for future work.

2 Theoretic Foundation

At this point, we will define the notation to be used in the article. Some typical terms from the fields of Machine Learning and Multiple Criteria Decision Making/Analysis will be considered interchangeable for the purpose of this study.

Let $A = \{a_1, ..., a_m\}$ be a set of m alternatives, and $G = \{g_1, g_2, ..., g_n\}$ be a set of n criteria. Let $\mathbf{X} \in \mathbb{R}^{m \times n}$ be a data matrix (decision matrix), where each row represents an alternative (object, example), and each column represents a criterion (attribute, dimension). In this study, the attribute vector related to an alternative a_i will be denoted as $x_i = [x_{i,1}, x_{i,2}, ..., x_{i,n}]$, which is also the i^{th} row of the matrix $\mathbf{X}$. The value of alternative a_i for criterion g_j is given by $x_{i,j}$, i.e., the entry at position (i, j) of the decision matrix $\mathbf{X}$.

2.1 K-means

The K-means algorithm divides the dataset into k clusters. Initially, k elements are randomly selected as the initial clusters' centroids, and distances from each instance of the dataset to each centroid are calculated. Each object is then assigned to the cluster whose centroid is closest to it. Next, the centroids are updated by recalculating them based on the clusters obtained in the previous step. Then, the distances from each object to the new centroids are calculated, and a new clustering is performed. This process is repeated until there is no further change in the allocation of objects to clusters or until another stopping criterion is observed. The K-means steps are organized in Algorithm 1.

Algorithm 1 K-means algorithm, based on [13]

Input: Dataset $\mathbf{X}$, number of clusters K.
Output: Partition of $\mathbf{X}$ into K groups.
1: Initialize K centroids by randomly choosing K instances of the Dataset (rows of $\mathbf{X}$).
2: **While** the stopping criterion is not met **do**
3:     **For each** object $\mathbf{x}_i \in \mathbf{X}$ **do**
4:         **For each** centroid $\overline{\mathbf{x}}_j$ **do**
            Calculate the distance $d(\mathbf{x}_i, \overline{\mathbf{x}}^{(j)})$
        end
          Assign $\mathbf{x}_i$ to the cluster C_j with the closest centroid.
    end
5: Update the centroids by calculating them from the clusters formed by the assignments.
    return The clusters and their centroids.

2.2 ELECTRE-III

Proposed by [22], the ELECTRE-III method considers the concept of pseudo-criterion, using indifference (q_j) and preference (p_j) thresholds to handle the preferences imposed by the decision maker(s). Algorithm 2 illustrates the main steps of the method. The construction phase of the outranking relation comprises Steps 1, 2, and 3, where concordance and discordance indices are calculated, and the credibility of the outranking relation between pairs of alternatives is evaluated. The subsequent steps correspond to the exploration of these relations, from which two complete pre-orders of the alternatives are obtained. To achieve this, two rules are used to order alternatives: descending and ascending.

Algorithm 2 ELECTRE-III Algorithm, adapted from Rogers (2000)

Require: The following inputs are required.

- $A = \{a_1, a_2, \dots a_n\}$, a set composed by n alternatives or objects to be ranked.
- $F = \{f_1, f_2, \dots f_m\}$, a family composed by m criteria or variablesused for building the ranking.
- G, a matrix that stores the performance $g_j(a_i)$ of each a_i under each criterion f_j.
- $G \in \mathbb{R}^{n\times m}$, a matrix by the scalars, so that g_{ij} stores the performance of the i^{th} alternative $a_i \in A$ under the j^{th} criterion $f_j \in F$.

$$G = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,m} \\ x_{2,1} & x_{2,2} & \dots & x_{2,m} \\ \vdots & \vdots & \dots & \vdots \\ x_{n,1} & x_{n,1} & \dots & x_{n,m} \end{bmatrix}$$

- W, a vector that contains the importance w_j of each criterion f_j for making the ranking.

1: Read the inputs A, F, G, and W.

2: Commpute of the concordance index ($C(a,b)$):

$$C(a,b) = \frac{\sum\limits_{j=1}^{\text{n}} w_j c_j(a,b)}{\sum\limits_{j=1}^{\text{n}} w_j}$$

$$c_j(a,b) = \begin{cases} 1, \text{ if } g_j(a) + q_j(g_j(a)) \geq g_j(b) \\ 0, \text{ if } g_j(a) + p_j(g_j(a)) < g_j(b) \\ \frac{g_j(a) - g_j(b) + p_j(g_j(a))}{p_j(g_j(a)) - q_j(g_j(a))}, \text{ otherwise} \end{cases}$$

3: Compute the discordance index ($D(a,b)$)

$$D_j(a,b) = \begin{cases} 0, \text{ if } g_j(b) \geq g_j(a) + p_j(g_j(a)) \\ 1, \text{ if } g_j(b) > g_j(a) + v_j(g_j(a)) \\ \frac{g_j(b) - g_j(a) - p_j(g_j(a))}{v_j(g_j(a)) - p_j(g_j(a))}, \text{ otherwise} \end{cases}$$

where v_j: veto threshold for criterion j

4: Calculate credibility degree of the outranking of b by a ($S(a,b)$)

$$S(a,b) = \begin{cases} C(a,b), \text{ if } D(a,b) \leq C(a,b), \forall j \\ C(a,b) \prod\limits_{j \in J(a,b)} \frac{(1 - D(a,b))}{1 - C(a,b)}, \text{ otherwise} \end{cases}$$

5: Compute the qualification score (λ_0) and the cutoff level (λ_1)

$$\lambda_0 = \max_{a,b \in A} \{S(a,b)\}$$

$$\lambda_1 = \lambda_0 - s(\lambda_0)$$

where $s(\lambda_0)$ is the discrimination threshold.

6: Obtain the descending and ascending rankings through an iterative process with updating the value of λ_1 and applying the procedures:

$$\overline{D}_1 = \{a \in A / q_A^{\lambda_1} = \overline{q}_A = \max_{x \in A} q_A^{\lambda_1}(x)\}$$

$$\underline{D}_1 = \{a \in A / q_A^{\lambda_1} = \underline{q}_A = \min_{x \in A} q_A^{\lambda_1}(x)\}$$

where $q_A^{\lambda_1}(a)$ is the qualification of alternative a in relation to the others; calculated as the number of alternatives outranked by a under a cutoff level λ_1 minus the number of alternatives that outrank a with the same cutoff level.

return

The ranking of the alternatives in A.

2.3 Monte Carlo Simulation in MCDM/A

The use of MCDM/A approach to model real problems involves a series of information that may contain uncertainty. For instance, the data used as intra-criteria evaluations of alternatives can be imprecise. Also, the application of a specific MCDM/A method may involve a process of preference elicitation, where the Decision-Maker (DM) should define some parameters such as weights and thresholds to be used. The definition of precise values for those parameters may also add a source of uncertainty in the decision-making process. A technique that stands out in the MCDM/A literature to deal with uncertainty is named Stochastic multicriteria acceptability analysis (SMAA), initially proposed by [14] and extended to SMAA-2 by [15].

To sum up, SMAA uses Monte-Carlo Simulation to compute the probability of each alternative to be most preferred or to be assigned to a particular rank or category. Monte Carlo Simulation is a powerful stochastic computing simulation approach used to model complex scenarios by employing the use of random sampling to obtain numerical solutions, see [24] for a detailed reference. The approach involves running numerous simulations to approximate the behavior of a situation. In the case of MCDM/A though SMAA, it is used to emulate the decision-making process to estimate uncertain outcomes given the uncertainty in the input data (evaluations and/or parameters). Therefore, different combinations of parameters are tested through the simulated runs, and so, accounting for possible imprecision in the parameter definitions and allowing the conduction of sensitivity analyses [18]. For instance, in ELECTRE-III, instead of considering a specific value for a preference threshold $p_j = 0.4$, one could define this parameter as a random variable that follows a probability distribution, let's say $U(0.35; 0.45)$.

The SMAA approach has been used combined with a vast number of MCDM/A methods, including several classic outranking applications. For instance, a SMAA procedure was proposed for ELECTRE III by [11] and other variants and applications considering this method are found in [6, 28]. [18] presents a recent systematic literature review that includes several SMAA methods and applications to analyze decision-making problems. In this article, the Monte Carlo method was used to estimate the probability each cluster centroid outranks or is outranked by the others. Specifically, 10000 runs of ELECTRE III were used where the weights, indifference, and preference thresholds of the criteria were sampled from defined probability distributions. Then, the results include the observed proportion of each possible binary preference relation from ELECTRE III.

3 A Prosperity Index Assessment

In this section, we apply the proposed hybrid approach to analyze 167 countries considering 12 indicators regarding the 2023 Legatum Prosperity Index [12]. This institute annually reports the evaluation of countries around the world in terms of sustainable development. Table 1 presents the criteria, the domain of each criterion ranges from 0 to 100, and the criteria directions are monotonically positive, meaning that higher values are preferred.

One can note that the list of criteria includes indicators regarding both economic and social well-being. Specifically, the institute divides them into three groups: Inclusive Societies (represented by g_1, g_2, g_3, and g_4), Open Economies (g_5, g_6, g_7, and g_8), and Empowered People (g_9, g_{10}, g_{11}, and g_{12}). This kind of application often presents conflicting criteria/variables. For instance, a country that shows high performance regarding market conditions not necessarily will be well-rated when economic inequality aspects are considered. Table 2 presents a subset of the initial Decision Matrix, including 10 countries alphabetically ordered. The complete dataset is available at the Legatum Institute website [12].

The obtained clusters were then ranked with ELECTRE-III outranking method. Table 4 presents the initially used parameters, which include the weights, indifference, preference, and veto thresholds of the criteria. Observe this initial configuration considered equal weights. Also, to be consistent in terms of the method, we selected the thresholds so that $q_j \leq p_j \leq v_j, \forall j$. At this point, see that the determination of all parameters was made in a deterministic way. Therefore, a unique result is expected from ELECTRE-III.

Table 5 presents the partial ranking obtained by the ELECTRE-III procedure. In this application, it can be observed that two centroids were considered indifferent by the ELECTRE-III method and shared the second position in the ranking. Cluster 4, which is ranked in the best position, is mainly composed of developed countries, mostly from the northern hemisphere, with some exceptions such as Uruguay, Chile, and Costa Rica. These nations show excellent

Table 1. Criteria Table

g_j	Legatum (Prosperity)	Criteria Domain	Criteria Direction
g_1	Safety and Security	0 - 100	max
g_2	Individual Freedom	0 - 100	max
g_3	Governance	0 - 100	max
g_4	Social Capital	0 - 100	max
g_5	Investment Environment	0 - 100	max
g_6	Business Conditions	0 - 100	max
g_7	Infrastructure and Market Access	0 - 100	max
g_8	Economic Quality	0 - 100	max
g_9	Housing Conditions	0 - 100	max
g_{10}	Health	0 - 100	max
g_{11}	Education	0 - 100	max
g_{12}	Natural Environment	0 - 100	max

Table 2. Decision Matrix

Country	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}
Afghanistan	20.93	31.02	29.47	31.17	30.25	42.01	29.67	33.75	39.74	50.91	27.11	44.11
Albania	74.9	61.59	48.44	47.47	55.17	54.87	61.57	45.44	76.17	73.95	70.07	58.64
Algeria	74.7	39.1	41.96	39.33	38.98	43.05	51.12	39.95	78.24	73.22	59.57	46.29
Angola	61.33	41.13	35.64	39.62	25.24	32.59	34.93	41.81	44.33	49.88	29.61	50.47
Argentina	69.72	76.19	49.52	63.3	49.45	45.28	55.01	41.86	82.08	74.45	69.25	60.41
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Venezuela	40.45	36.63	13.09	58.3	21.69	20.5	43.34	26.44	69.82	69.61	61.1	62.22
Vietnam	69.1	34.85	47.86	65.97	45.24	52.52	63	60.33	71.8	76.99	66.56	52.09
Yemen	22.6	25.3	18.2	38.44	22.76	33.12	30.93	28.81	41.58	57.45	28.12	44.49
Zambia	66.05	48.29	42.03	50.63	43.2	52.92	35.98	30.93	40.82	57.2	39.04	58.6
Zimbabwe	63.48	37.91	31.66	45.82	28.09	41.03	38.89	37.74	47.34	55.46	56.03	52.53

performances in all pillars. In the last position, Cluster 2 is mainly composed of countries from the Middle East and Africa that still face many issues related to security, access to healthcare, and education. Figure 1 illustrates the clusters obtained for the dataset.

4 Results

All the experiments were performed using the Python programming language. The scikit-learn library [17] was used for normalization and the K-means procedure. Also, the PyDecision library [19] was used as implementation for the ELECTRE-III method. Also, we verified the clustering and ranking results with, respectively, Visual Clustering [8] and Visual Outdeck [7]apps. Finally, the post-analysis experiments used random numbers generated with the NumPy.

As a post-analysis of the obtained results, a study of the robustness of the rankings was conducted. For this purpose, 10,000 simulations of ELECTRE-III

Table 3. Cluster centroids

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}
cluster 1	0,67	0,65	0,50	0,56	0,51	0,53	0,56	0,49	0,74	0,72	0,64	0,53
cluster 2	0,43	0,38	0,28	0,37	0,18	0,35	0,14	0,21	0,26	0,36	0,19	0,41
cluster 3	0,66	0,23	0,42	0,52	0,57	0,58	0,63	0,61	0,79	0,79	0,68	0,36
cluster 4	0,88	0,84	0,83	0,71	0,88	0,83	0,83	0,82	0,96	0,88	0,89	0,75
cluster 5	0,51	0,34	0,33	0,44	0,33	0,42	0,36	0,33	0,56	0,60	0,42	0,38

Table 4. ELECTRE-III parameters

Parameter	Value
q_j, $\forall j \in \{1, ..., 12\}$	0.1
p_j, $\forall j \in \{1, ..., 12\}$	0.2
v_j, $\forall j \in \{1, ..., 12\}$	0.7
w_j, $\forall j \in \{1, ..., 12\}$	$\frac{1}{12}$

were performed, varying the input parameters of the model (weights, indifference thresholds, and preference thresholds). The weights followed a Dirichlet distribution, which is defined as a multivariate distribution over the weights (w_j) that ensures $\sum_{j=1}^{n} w_j = 1$ and $w_j \geq 0, \forall j \in \{1, ..., 12\}$. The for all criteria, indifference thresholds followed a Uniform distribution $U(0.07, 0.13)$, while the preference thresholds were sampled from a uniform distribution with values between 0.17 and 0.23 (i.e., $U(0.17, 0.23)$). Note that the definition of the distributions prevents inconsistent cases where $q_j \geq p_j$ for any criterion g_j.

For each simulation, the binary relationship between each pair of alternatives (clusters) was obtained. Table 6 presents the frequency at which each result is found in the simulations, and the robustness of the solutions can be confirmed. For example, cluster 4, the highest-ranked cluster, presents a P+ binary relation compared to the others in all the simulations. Observe that cluster 2, which was ranked in the last position in Table 5, was outranked by the other centroids in most of the runs, being considered indifferent to the centroid 4 in only 0.03% of the simulations. Also, it is worth noting that the indifference found in Table 5 between clusters 1 and 3 was confirmed in the majority of the generated simulations. In 66.07% of the cases, there was indifference between the alternatives. Considering the other simulations, cluster 1 had an advantage in 33.92% of the cases, while being outperformed by cluster 3 in only 0.01% of the tested scenarios with the distributions defined in this post-analysis.

	cluster 1	cluster 2	cluster 3	cluster 4	cluster 5	Posição
cluster 1	-	P+	I	P-	P+	2º
cluster 2	P-	-	P-	P-	P-	5º
cluster 3	I	P+	-	P-	P+	2º
cluster 4	P+	P+	P+	-	P+	1º
cluster 5	P-	P+	P-	P-	-	4º

Table 5. Preference relationships

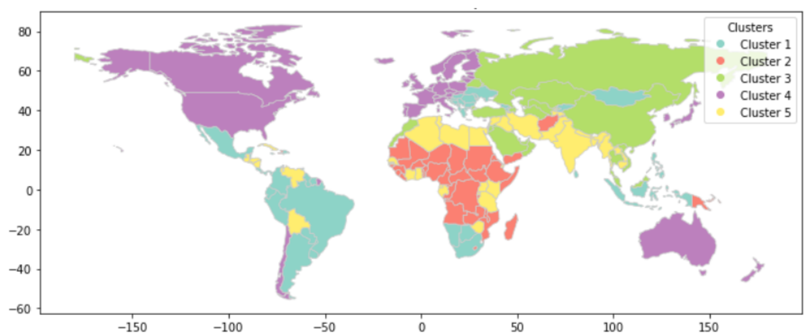

Fig. 1. Agrupamento de Países - Índice de Prosperidade

Table 6. Results from simulation

	cluster 1	cluster 2	cluster 3	cluster 4	cluster 5
cluster 1	-	P+: 100%	P+: 33.92% P-: 0.01% I: 66.07%	P-: 100%	P+: 100%
cluster 2	P-: 100%	-	P-: 100%	P-: 99.97% I: 0.03%	P-: 100%
cluster 3	P+: 0.01% P-: 33.92% I: 66.07%	P+: 99.97% I: 0.03%	-	P-: 100%	P+: 99.91% P-: 0.06% I: 0.03%
cluster 4	P+: 100%	P+: 100%	P+: 100%	-	P+: 100%
cluster 5	P-: 100%	P+: 97.85% P-: 0.24% I: 1.91%	P+: 0.06% P-: 99.91% I: 0.03%	P-: 100%	-

5 Conclusions

This article presents the application of the K-means algorithm together with the ELECTRE-III MCDM/A and the Monte Carlo method to analyze a set of 167 countries regarding 12 prosperity indicators. Initially, K-means was applied to group countries based on their similarities. Then, the centroids of the clusters were ranked using ELECTRE-III. Thus, additional information is obtained about the groups formed by the Unsupervised Learning procedure.

The results achieved were consistent with expectations. Initially, a deterministic set of parameters was considered and ELECTRE-III could order the clusters based on the performance of the centroids.

Aiming to explore the uncertainties related to the parameters of the modelling, a robustness analysis was conducted by simulating variations in ELECTRE-III parameters according to probability distributions, generating results for 10,000 simulations. This analysis confirmed the consistency of the obtained rankings and the binary relationships resulting from the ELECTRE-III method, as presented in Table 6.

The application also illustrated how an MCDM/A outranking approach may add information to the clustering technique. The use of ELECTRE-III also makes it possible to consider the variables as pseudocriteria through the use of thresholds that account for possible hesitation of the Decision-Maker when determining his/her preferential information.

5.1 Further issues

We suggest to investigate the combination of our proposal with fuzzy ELECTRE (see [1], and also to compare our results against those that should be gotten trough dynamic programming and genetic algorithms. Beyond this direction, future work can apply this approach to different datasets, as well as make comparisons using different procedures, either in the data clustering step using different unsupervised algorithms or in the clustering ranking step using other MCDM/A methods. In addition, the results of the hybrid approach may be compared to MCDM/A sorting techniques and to other methods capable of obtaining ordered clusters.

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References

1. Azadnia, A.H.: Supplier selection: a hybrid approach using electre and fuzzy clustering. In: Informatics Engineering and Information Science: International Conference, ICIEIS 2011. Springer Berlin Heidelberg, Kuala Lumpur, Malaysia (2011)
2. Azevedo, G.D.C., Costa, H.G.: Applying non-compensatory multicriteria methods to build better life index countries ranking. In: 2022 IEEE LATIN AMERICAN CONFERENCE ON COMPUTATIONAL INTELLIGENCE (LA-CCI). pp. 53–58 (2022). https://doi.org/10.1109/LA-CCI54402.2022.9981854, iEEE Latin American Conference on Computational Intelligence (LA-CCI), Montevideo, URUGUAY, NOV 23-25, 2022

3. Barroso, J.S.S., Araujo, E.A.: The trajectory of the sri fund industry in brazil. Global Business and Economics Review **27**(4), 393–405 (2022)
4. do Carvalhal Monteiro, R.L., Pereira, V., Costa, H.G.: Analysis of the better life index trough a cluster algorithm. Social Indicators Research **142**, 477–506 (2019)
5. Chowdhury, P., Paul, S.K.: Applications of MCDM methods in research on corporate sustainability. Management of Environmental Quality: An International Journal **31**(2), 385–405 (feb 2020). https://doi.org/10.1108/MEQ-12-2019-0284, https://www.emerald.com/insight/content/doi/10.1108/MEQ-12-2019-0284/full/html
6. Corrente, S., Figueira, J.R., Greco, S., Słowiński, R.: A robust ranking method extending electre iii to hierarchy of interacting criteria, imprecise weights and stochastic analysis. Omega **73**, 1–17 (2017)
7. Costa, H.G.: Visual OutDecK: A Web APP for Supporting Multicriteria Decision Modelling of Outranking Choice Problems. In: Abraham, A., Hong, T.P., Kotecha, K., Ma, K., Manghirmalani Mishra, P., Gandhi, N. (eds.) Hybrid Intelligent Systems. pp. 907–916. Springer Nature Switzerland, Cham (2023)
8. Costa, H.G., Costa, L.F.: A no-code web app for supporting decision modelling of clustering problems. In: International Conferences on Applied Computing 2023, AC 2023 and WWW/Internet 2023, ICWI 2023. pp. 222 – 226. IADIS Press, Funchal, Ilha da Madeira, Portugal (2023), https://www.iadisportal.org/ac-icwi-2023-proceedings
9. García-Martínez, G., Guijarro, F., Poyatos, J.A.: Measuring the social responsibility of European companies: A goal programming approach. International Transactions in Operational Research **26**, 1074–1095 (2019). https://doi.org/10.1111/itor.12438
10. Greco, S., Ehrgott, M., Figueira, J.R. (eds.): Multiple Criteria Decision Analysis, International Series in Operations Research & Management Science, vol. 233. Springer New York, New York, NY, 2 edn. (2016). https://doi.org/10.1007/978-1-4939-3094-4, http://link.springer.com/10.1007/978-1-4939-3094-4
11. Hokkanen, J., Lahdelma, R., Miettinen, K., Salminen, P.: Determining the implementation order of a general plan by using a multicriteria method. Journal of Multi-Criteria Decision Analysis **7**(5), 273–284 (1998)
12. Instituto Legatum: The Legatum Prosperity Index: Creating the Pathways from Poverty to Prosperity. Tech. rep., Legatum Institute (2023), https://www.prosperity.com/rankings
13. James, G., Witten, D., Hastie, T., Tibshirani, R., Taylor, J.: An Introduction to Statistical Learning. Springer Texts in Statistics, Springer International Publishing, Cham (2023). https://doi.org/10.1007/978-3-031-38747-0, https://link.springer.com/10.1007/978-3-031-38747-0
14. Lahdelma, R., Hokkanen, J., Salminen, P.: Smaa-stochastic multiobjective acceptability analysis. European Journal of Operational Research **106**(1), 137–143 (1998)
15. Lahdelma, R., Salminen, P.: Smaa-2: Stochastic multicriteria acceptability analysis for group decision making. Operations research **49**(3), 444–454 (2001)
16. Ozmen, M.: Logistics competitiveness of oecd countries using an improved todim method. SADHANA-ACADEMY PROCEEDINGS IN ENGINEERING SCIENCES **44**(5) (MAY 2019). https://doi.org/10.1007/s12046-019-1088-y
17. Pedregosa, F., Varoquaux, G., Gramfort, A., Michel, V., Thirion, B., Grisel, O., Blondel, M., Prettenhofer, P., Weiss, R., Dubourg, V., Vanderplas, J., Passos, A., Cournapeau, D., Brucher, M., Perrot, M., Duchesnay, E.: Scikit-learn: Machine learning in Python. Journal of Machine Learning Research **12**, 2825–2830 (2011)

18. Pelissari, R., Oliveira, M.C., Amor, S.B., Kandakoglu, A., Helleno, A.L.: Smaa methods and their applications: a literature review and future research directions. Annals of Operations Research **293**, 433–493 (2020)
19. Pereira, V.: pyDecision (2023), https://github.com/Valdecy/pyDecision/
20. Rani, V., Nabi, S.T., Kumar, M., Mittal, A., Kumar, K.: Self-supervised learning: A succinct review. ARCHIVES OF COMPUTATIONAL METHODS IN ENGINEERING (2023). https://doi.org/10.1007/s11831-023-09884-2
21. Raschka, S., Liu, Y.H., Mirjalili, V., Dzhulgakov, D.: Machine Learning with PyTorch and Scikit-Learn: Develop machine learning and deep learning models with Python. Packt Publishing Ltd (2022)
22. Roy, B.: Electre iii: Algorithme de classement basé sur une représentation floue des préférences en présence de critères multiples. Cahiers du CERO **20**, 3–24 (1978)
23. Roy, B.: Multicriteria methodology for decision aiding. Springer New York, New York (1996). https://doi.org/10.1007/978-1-4757-2500-1
24. Rubinstein, R.Y., Kroese, D.P.: Simulation and the Monte Carlo method. John Wiley & Sons (2016)
25. Siksnelyte-Butkiene, I., Streimikiene, D.: Sustainable development of road transport in the eu: Multi-criteria analysis of countries' achievements. ENERGIES **15**(21) (NOV 2022). https://doi.org/10.3390/en15218291
26. de Souza, R.L.S., Costa, H.G.: K-means and multicriteria decision aid applied to sustainability evaluation. In: Abraham, A., Piuri, V., Gandhi, N., Siarry, P., Kaklauskas, A., Madureira, A. (eds.) Intelligent Systems Design and Applications. pp. 1198–1208. Springer International Publishing, Cham (2021). https://doi.org/10.1007/978-3-030-71187-0_111, https://link.springer.com/10.1007/978-3-030-71187-0_111
27. Stojčić, M., Zavadskas, E., Pamučar, D., Stević, Ž., Mardani, A.: Application of MCDM Methods in Sustainability Engineering: A Literature Review 2008–2018. Symmetry **11**(3), 350 (mar 2019). https://doi.org/10.3390/sym11030350, https://www.mdpi.com/2073-8994/11/3/350
28. Zhou, H., Wang, J.q., Zhang, H.y.: Stochastic multicriteria decision-making approach based on smaa-electre with extended gray numbers. International Transactions in Operational Research **26**(5), 2032–2052 (2019)