Evidential linear regression for soft Detrended Fluctuation Analysis

Nicolas Sutton-Charani[1][0000−0002−3065−0712] and Francis Faux[2][0000−0001−9272−8215]

[1] Euromov Digital Health in Motion, Univ Montpellier, IMT Mines Ales, Montpellier, France
[2] IRIT, CNRS & Université de Toulouse, France,
nicolas.sutton-charani@mines-ales.fr, francis.faux@irit.fr

Abstract. Detrended Fluctuation Analysis (DFA) provides insights on signal complexity which have shown to be relevant and effective for distinguishing healthy and non-healthy persons through different physiological signals. This method is based on different steps involving linear regression. This paper proposes an evidential linear regression model and its application on DFA in order to take into account limitations of DFA due to uncertainties associated with crisp linear regression estimates. Three R packages that contains a DFA implementation has been compared with the proposed method in experiments realised on sinusoidal signals, noises and Hausdorff famous dataset that illustrates the interest of the method.

Keywords: Complexity · Fractals · Uncertainty · Regression · Belief functions.

1 Introduction

In fractal geometry, a fractal object appears exactly or approximately similar to a part of itself at various scales, it is composed of sub-units [9]. By analogy, this property of spatial self similarity has been extended to time series. Indeed, the fractal concept can be applied not just to irregular geometric forms that lack a characteristic scale of length, but also to certain complex processes that have no single time scale [17, 19, 12]. In this case, the self-similarity is expressed in terms of statistical properties (e.g., the mean and standard deviation of a time series segment are scaled versions of the mean and standard deviation of the whole) and the fractal object can then be viewed as a statistical fractal [13].

The presence of temporal fractal similarity can naturally be observed on stochastic signals (fractional Gaussian noise (fGn), pink noise, ...) with hidden properties also referred to as long-range correlations, long-range dependence or long term memory. It is also one critical marker of physiological systems characterised by their extraordinary complexity [2]. Indeed complexity is often associated with a certain level of coordination of a great number of interactions in a multiple-component system [5]. The presence of temporal fractal properties are widespread in humans across heart rate variability [11], gait cycles of human walking [20], brain activity, autonomic control, etc.

Among many existing methods to estimate the self-similarity in time series, the detrended fluctuation analysis (DFA), introduced by Peng et al. [18], has been preferred because of its adaptability to non-stationary signals. The DFA computes the root mean square error of linear fits realised on signals windows of increasing size and gives a single exponent as a result. However, several authors have mentioned the need for complexity levels expressed by intervals rather than single values. For example Goldberger [12] says that in an ideal world, the property of self similarity holds on all scales whereas the real world, however, necessarily imposes upper and lower bounds. Setty [22] mentions that standard DFA cannot distinguish between multi-fractal and mono-fractal signals as it gives a single exponent as a result. Bryce [3] indicates that DFA imparts serious biases for short data sets, thus requiring caution in the interpretation of the estimated value. Rojjo-Alvarez *et al.* highlighted high uncertainties about long term correlations in DFA physiological interpretation. Moreover many arbitrary choices have been proposed about the framing of fluctuations computation (sliding windows, growing factor of windows length, etc.). This leads to different DFA implementations with different and sometimes inconsistent results.

In order to address some of these limitations, this paper proposes a new soft DFA model based on a new imprecise linear regression model which results in an evidential representation of temporal complexity. Two kinds of uncertainties related to the considered windows sizes are taken into account in the model. First the uncertainty in the linear regression due to the dilemma between linear fluctuations (residuals) and second, the number of points (reliability). From a practical point of view, since many DFA designs (framing, fluctuations aggregation, etc.) and implementations have been proposed, in this paper the following R packages are compared in terms of DFA behavior: *nonlinearTseries*[1], *DFA*[2] and *casnet*[3].

The organisation of the rest of the paper is as follows. In Section 2 we recall the necessary basis of the DFA for time series complexity estimation. Section 3 presents a soft approach of the DFA. Experiments and results are detailed in Section 4 and finally we present our concluding remarks and perspectives.

2 Background

2.1 Detrended Fluctuation Analysis (DFA) overview

DFA [18] is a method for analysing scaling behaviour and testing for self similarity in time series as it measures the dispersion of the residuals of linear fluctuations regressed at different terms. The basic algorithm of DFA consists in three steps [3] :

[1] https://cran.r-project.org/web/packages/nonlinearTseries
[2] https://cran.r-project.org/web/packages/DFA
[3] https://rdrr.io/github/FredHasselman/casnet

1. a raw time series $x = (x_i)_{i=1,...,N}$ is first decentered and integrated such that the resulting time series is $X = (X_i)_{i=1,...,N} = \left(\sum_{j=1}^{i}(x_j - \bar{x})\right)_{i=1,...,N}$ where $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$. The time series $(X_i)_{i=1,...,N}$ is then segmented into several windows of increasing size n.
2. for each window size n, K windows $n_k = (X_k, ..., X_{k+n})$ for $k = 1, ..., K$ are considered, for each of them X is locally fit to a linear regressor $X^{k,n}$ (eventually extended to a polynomial one) and the mean square residual $F(k,n)$ is defined such that:

$$F(k,n) = \sqrt{\frac{1}{n}\sum_{j=k}^{k+n}[X_j - X_j^{k,n}]^2} \tag{1}$$

3. finally for each window size n, the average square residual $F(n)$ is computed on all windows of size n:

$$F(n) = \frac{1}{K}\sum_{k=1}^{K} F(k,n) \tag{2}$$

The main DFA hypothesis is that the detrended fluctuation are supposed to grow exponentially in regards to the window size, i.e., $F(n) = C.n^{\alpha}$ where C is constant and α is the complexity level of x. By considering the logarithm of both terms we get $\log F(n) = \log C + \alpha \log n$. The coefficient α can then be estimated through a linear regression of the log-fluctuations given the log-terms $log(n)$. The scaling exponent α can also be interpreted as an estimate of the Hurst exponent [15] with value giving information about the series self-correlations:

- $0 < \alpha < 0.5$ means the process has a memory and it exhibits anti-correlations. We have $\alpha \approx 0.1$ for random process called a blue noise whose power spectral density increases proportionally to frequency f over a finite frequency range.
- $\alpha = 0.5$ means the process is indistinguishable from a random process with no memory and thus no correlation. We have $\alpha \approx 0.5$ for a random process called a white noise whose power spectral density is constant at all frequencies f.
- $0.5 < \alpha < 1$ then the process has a memory, and it exhibits positive correlations. We have $\alpha \approx 1$ for a random process called a pink noise whose power spectral density is inversely proportional to the frequency f of the process.
- $1 < \alpha < 2$ means the process is non-stationary. We have $\alpha \approx 1.5$ for a random process called a Brownian noise whose power spectral density is inversely proportional to f^2, meaning that it has a higher intensity at low frequencies, even more so than pink noise.

2.2 Limitations of DFA

Many open questions remain about the DFA. At the methodological level DFA is known to display significant curvature on log-log plots for short time windows which emphasises short-terms non-linearity (noise) [3]. Some studies show that DFA yields a single Hurst exponent, and thus cannot distinguish between multi-fractal and mono-fractal systems [22]. For short data sets previous studies show sensitive impact of data size used on the Hurst exponent, thus requiring caution in the interpretation of the estimated value [16]. Nevertheless, the fidelity of DFA complexity estimation strongly depends on the windowing design and on arbitrary choices in parameters tuning in an unclear manner. The choice of averaging the signal fluctuations (see Eq. 2) necessarily implies a potentially significant information loss. Finally, the DFA interpretation remains unclear and partially reliable due to the difficulty of clearly demonstrating the underlying physiological process. All these questions cause different types of uncertainty on the DFA process.

3 Soft approach

The method presented in this section proposes an evidential extension of the DFA that uses the residuals' distribution of the fluctuations-terms to determine an imprecise regression model.

3.1 Imprecise regression

Many imprecise or uncertain regression methods have been proposed in the literature. Some of them are related to the Bayesian framework [23] with prior distribution on the regression parameters (generally provided by domain experts which are not always available), others concern machine learning adaptations to uncertain data [10, 4], others try to benefit from uncertainty modelling inside models [1, 7] and still some others deal with models predictions uncertainty or confidence quantification [21, 8]. Most of these approaches are neural-network based and many of them involve likelihood extensions and estimations but only a few are defined in the linear context.

In the DFA, the complexity α is computed as the slope of the linearly log-regressed fluctuations. The standard DFA complexity is computed on precise fluctuations without dealing with their associated uncertainty. The imprecise linear regression model proposed in this paper considers the uncertainty inside models and in output predictions.

3.2 Evidential linear regression

The idea of the imprecise regression model proposed in this paper is to use all the regression residuals in order to soften the final regression steps of the DFA and to output a complexity belief function. The focal elements of the output belief mass are composed of complexity intervals. The regression model is defined in the uni-variate context, from a set of points $D = (x_i, y_i)_{i=1,\dots,n}$. The slope a and the intercept b of the linear regression line $y = ax + b$ are estimated from D, generally according to a least squares minimisation of residuals.

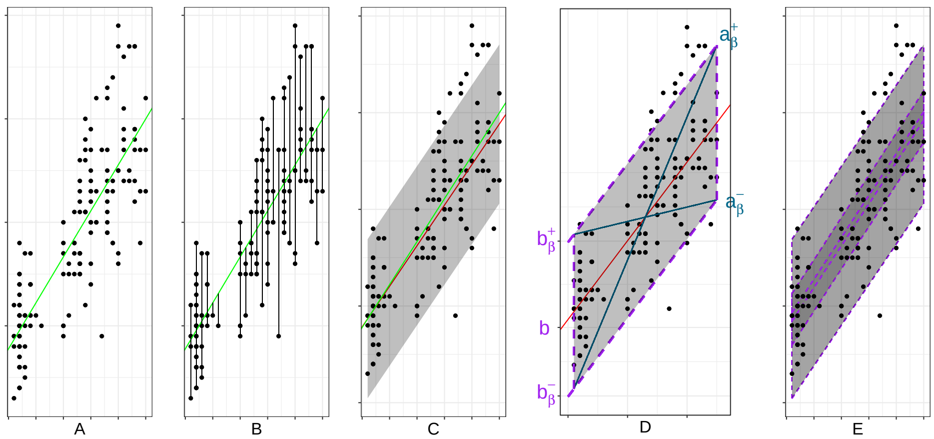

Fig. 1. Evidential linear regression on Iris dataset.

Evidential bands

Given a belief degree $\beta_k \in [0, 1]$, an *evidential band* is defined as the area between 2 (top and bottom) lines parallel to the precise regression line computed on the whole xy-scatter plot. These 2 lines are vertically set such that the band contains a proportion β_k of the closest points to the regression line (see Fig 1.A, B and C). From the evidential band, the slope interval $[a^-_{\beta_k}, a^+_{\beta_k}]$ and the intercept interval $[b^-_{\beta_k}, b^+_{\beta_k}]$ can be computed respectively according to the 2 band diagonals $a^-_{\beta_k}$ and $a^+_{\beta_k}$ and to the band top an bottom intercepts $b^-_{\beta_k}$ and $b^+_{\beta_k}$ (see Fig 1.D). It can be noted in Fig 1.C that, inside the evidential band, the new regression line is a corrected version of the initial one (after outliers removal). This implies that the different evidential bands (corresponding to different belief degrees) might not be parallels.

Imprecise regression model

From the above definition of evidential band, a global evidential linear regression model can be derived by considering sets of evidential bands associated with their belief degrees $\beta = \{\beta_1, ..., \beta_K\}$ in order to define 2 mass functions m^β_a and m^β_b related to the regression slope and intercept. The frame of discernment $\mathbb{R}$ is then added to the masses focal elements as a safety component with a mass assignment of $\frac{1}{n+1}$ inspired by an evidential generative model proposed by Dempster in [6]. Two mass functions m^β_a and m^β_b are finally defined according to the belief degrees $(\beta_1, ..., \beta_K)$ which are all associated with slope intervals $[a^-_{\beta_k}, a^+_{\beta_k}]$ and intercept intervals $[b^-_{\beta_k}, b^+_{\beta_k}]$. The degrees of belief β are finally normalised into 2 mass functions m^β_a and m^β_b such that:

$$\begin{cases} m_a^\beta([a_{\beta_1}^-, a_{\beta_1}^+]) &= \frac{\beta_1}{\frac{(n+1)}{n} \cdot \sum_{k=1}^{K} \beta_k} \\ \quad \vdots \\ m_a^\beta([a_{\beta_K}^-, a_{\beta_K}^+]) &= \frac{\beta_K}{\frac{(n+1)}{n} \cdot \sum_{k=1}^{K} \beta_K} \\ m_a^\beta(\mathbb{R}) &= \frac{1}{n+1} \end{cases} \tag{3}$$

and

$$\begin{cases} m_b^\beta([b_{\beta_1}^-, b_{\beta_1}^+]) &= \frac{\beta_1}{\frac{(n+1)}{n} \cdot \sum_{k=1}^{K} \beta_k} \\ \quad \vdots \\ m_b^\beta([b_{\beta_K}^-, b_{\beta_K}^+]) &= \frac{\beta_K}{\frac{(n+1)}{n} \cdot \sum_{k=1}^{K} \beta_k} \\ m_b^\beta(\mathbb{R}) &= \frac{1}{n+1} \end{cases} \tag{4}$$

The focal elements of m_a^β and m_b^β contain the different slope and intercept intervals and the real line (for security). In our implementation[4] we consider by default a 3-dimensional uncertainty granularity: $(\beta_1, \beta_2, \beta_3) = (10\%, 50\%, 95\%)$ (as in Fig. 1.E). Those default values represent a light illustrative (3 values) modelling choice covering almost the whole credibility ranges, but can be optimised according to different criteria depending on the application context and objectives. Depending on the desired expressiveness granularity of the evidential complexity, different belief degree sets β can be used.

3.3 Soft DFA

Our first contribution in terms of DFA is to keep uncertainty information by avoiding averaging signals fluctuations by term. In our *custom* precise DFA implementation, we consider a fixed (10 by default) number of windows for each signal size n. These windows are placed randomly on the signal and we use all the linear residuals computed in them for the final regression step. A preliminary analysis was done that showed no significative difference on standard DFA when removing the fluctuations averaging step.

The idea of the soft DFA proposed in this paper is to apply our evidential linear regression model on the log-fluctuations/terms data now enhanced. The result of the soft DFA is thus composed of a belief mass on the signal complexity level (Eq. 3). Our implementation of the proposed custom and soft DFA are available in our github[5].

In Fig. 2, the custom (left) and soft (middle and right) DFA results are represented on the enriched log-fluctuations dataset. The belief bands do not appear to be parallel which shows the ability of the soft DFA to use the residuals distribution in order to correct the slope estimate.

4 Experiments

In order to compare the soft DFA proposed in this paper to existing usual DFA implementations, 3 different complexity analysis contexts are considered in this section. First, the complexity of simple sinusoidal signals and noises are studied and finally some results of a previous study of Hausdorff *et al.* [14] are partially replicated with different DFA implementations including our soft approach.

[4] https://github.com/sutton-charani/possibilistic_linear_regression
[5] https://github.com/sutton-charani/soft_DFA

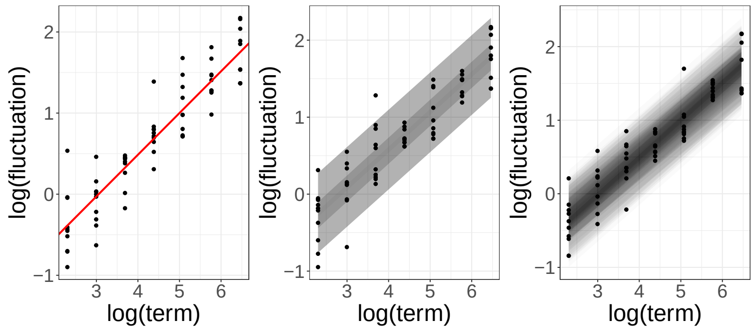

Fig. 2. Soft DFA on a Gaussian noise with belief levels $\beta = (10\%, 50\%, 95\%)$.

4.1 Basics signals

DFA on a sinusoidal signal In this experiment the properties of 3 deterministic sinusoidal signals are studied through DFA analysis.

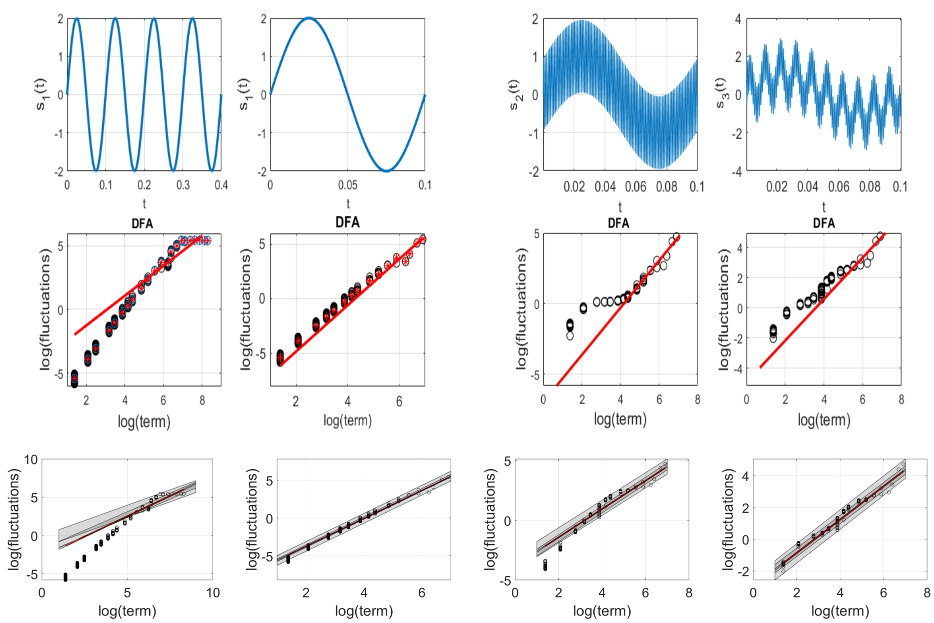

Fig. 3. DFA applied respectively to the signal s_1 for 4 periods (column 1), the signal s_1 limited to one period (column 2), the signals s_2 (column 3) then s_3 (column 4)

The figure 3 presents the results first for a simple sinusoidal signal $s_1(t) = \sin(2\pi f_1 t)$ for 4 periods (column 1) then limited to one period (column 2), for the sum of 2 sinusoids $s_2(t) = \sin(2\pi f1t) + \sin(2\pi f3t)$ (column 3) and finally for the sum of 3 sinusoids $s_3(t) = \sum_{i=1}^{3} \sin(2\pi f_i t)$ (column 4) with $f_1 = 10Hz$,

$f_2 = 100Hz$, and $f_3 = 1000Hz$. For the various signals, the second row of the figure 3 shows the residuals of the regression function of window size on a logarithmic scale. The last row presents the results of the soft DFA model using a three belief levels: $\beta = (\beta_1, \beta_2, \beta_3) = (0.1, 0.55, 0.95)$.

By observing the results of the DFA applied on s_1 for 4 periods (row 2, column 1), we can see that the residuals are quasi linear up to the size window $\log(term) = 7$ then a discontinuity is observed. Hence, the precise regression (in red) does not accurately reflect the behavior of DFA and appears biased and unreliable. Conversely when considering only a single period (column 2), the DFA exhibits a quasi-linear behavior and we notice that the soft DFA model fits the residual curve. This result highlights the linearity of the sinusoidal signal log-fluctuations when limited to one period.

For the additive combination of sinusoids (signals s_2 and s_3), the signals log-fluctuations are clearly non-linear. For the signal s_3, the soft DFA with a confidence level of 0.95 encompasses all residuals and may allows to define a reliable but imprecise degree of fractal complexity unlike the signal s_2 whose short terms log-fluctuations appear to grow slower than long terms ones. Table 1 presents the value of α for the precise DFA (column 2) and the imprecise range of α values (columns 3,4,5) for the soft DFA model observed in the third row of the figure 3.

		Confidence intervals								
		0.1			0.5			0.95		
Signal	precise α	α_{min}	α_{max}	m	α_{min}	α_{max}	m	α_{min}	α_{max}	m
s_1 for 4 period	1.07	0.98	1.14	0.0644	0.70	1.23	0.322	0.36	1.24	0.612
s_1 1 period	1.84	1.81	1.88	0.0644	1.77	1.94	0.322	1.62	2.09	0.612
s_2	1.19	1.15	1.22	0.0644	1.05	1.35	0.322	0.86	1.44	0.612
s_3	1.05	1.03	1.07	0.0644	0.93	1.14	0.322	0.84	1.29	0.612

Table 1. Values of α for the precise and the soft DFA models applied to s_1, s_2, s_3 and observed on the figure 3, third line

Noises Since the fractal complexity levels of a signal can be interpreted as noise components weights (see Subsection 2.1), and since DFA presents known limitations on small signal [3], different white, pink and brown small signals of different sizes n have been simulated and analysed with different R implementations including the precise (*custom*) and soft DFA methods proposed in this paper.

In Fig. 4, we observe important complexity biases for the *DFA* and *casnet* packages. Indeed, the light blue line (*DFA* package) appears shifted from the target interpretation α values (0.5, 1 and 1.5 respectively for white, pink and brown noises) especially for white and brown noise. The light green line (*casnet* package) shows α overestimates mainly for white noise (this package requires a minimum of 70 points to compute α). The *nonlinearTimeseries* package and our *custom* DFA implementation show no significant biases, even for short signals. The root mean squared errors (RMSE) have been computed for each implementation in regards to the target α values (see Table 2). For the soft DFA approach

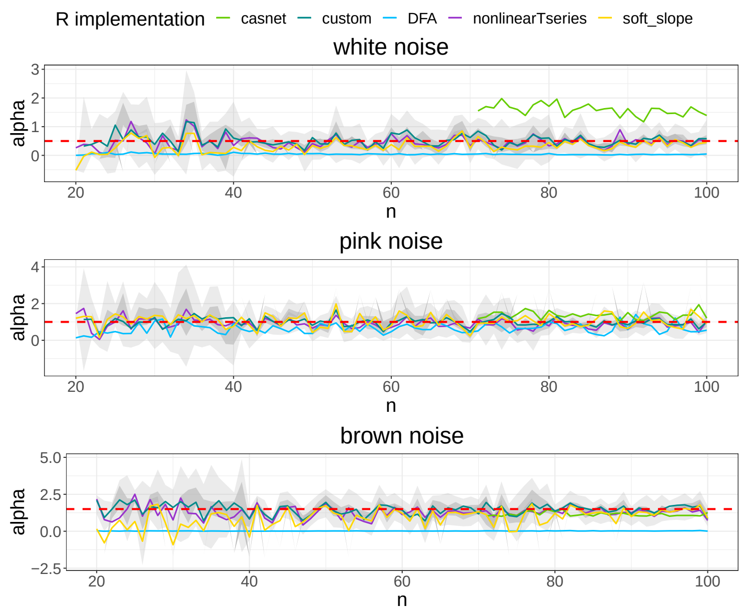

Fig. 4. DFA (α) sensitivity to simulated noises size (n).

the RMSE was computed on the resulting α pignistic expectation. The best α estimates (RMSE=0.27) was obtained with our *custom* DFA implementation (no fluctuation averaging and 10 uniformely random framings). The *nonlinearTimeseries* package provided the second best α estimates with an RMSE of 0.33. The lightening of our proposed custom DFA does not seem to cause interpretation biases or reliability decrease. The soft DFA observed RMSE was of 0.43 which suggests that the pignistic transform might not be the best decision rule in the DFA context. The *casnet* and *DFA* estimates show high errors (RMSE=0.63 and 0.8) probably due to previously mentioned biases which highlights the critical impact of the DFA preprocessing (framing, etc.)

nonlinearTimeseries	DFA	casnet	custom	soft DFA
0.33	0.80	0.63	0.27	0.43

Table 2. RMSE of α estimates of noises complexity

The soft DFA (Section 3.3) was applied on the previously described 3 noise signals and it can be seen in Fig. 4 that the evidential bands width decrease when the signal length increases. This is coherent with the desired property of DFA reliability negatively correlated with signals length as the amount of available information increases the confidence one can have about a DFA.

4.2 Hausdorff results replication

One of the first and most famous work on DFA was proposed by Hausdorff *et al.* in 2002 (Fig 3 and 4 of [14]) where humain gait rhythm was analysed through the

DFA perspective. The authors showed among other things that gait complexity was higher for very young children than for teenagers which highlights the physiological neuro-plasticity loss during human aging. The dataset was made available by the authors. In this section we tried to replicate this result by mea-

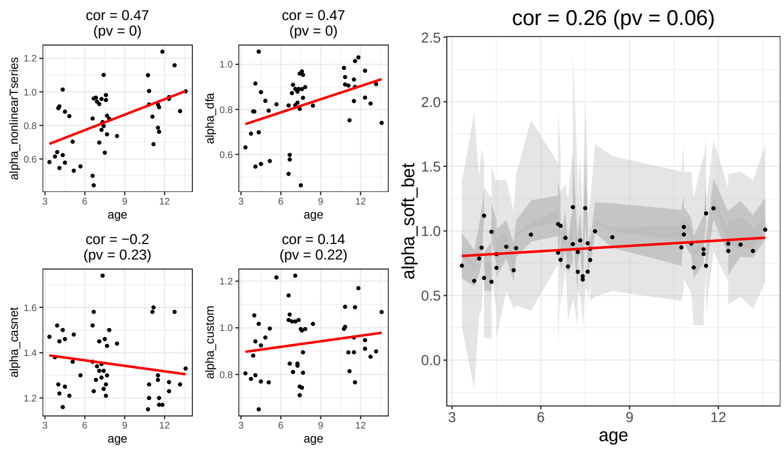

Fig. 5. Hausdorff results replication with precise and soft DFA.

suring the correlation between children ages and their gait complexity (α). With the 4 precise implementations considered in this paper (R packages *nonlinearTseries*, *DFA* and *casnet*, and our *custom* implementation), the corresponding scatter plots and regression lines wererepresented in the left part of Fig. 5 as well as the associated Pearson correlation coefficients and their p-values (Pearson correlation test) at the top of each plot. On the right part of Fig. 5 the same representation is proposed for the soft DFA with evidential bands computed for each points and a regression line corresponding to the pignistic expectation (the correlation coefficient and the associated p-values are computed according to this precise complexity estimator) computed from the soft DFA evidential bands.

It is noticeable that the *nonlinearTseries* and *DFA* package result in a positive (and significant) correlation between children ages and their gait complexities. This is in contradiction with Hausdorff results (Fig 3 and 4 of [14]) and common knowledge. Indeed, the fractal complexity of human physiological signals is known to decrease with age or diseases. In this context, this highlights a non-reproducibility issue with DFA studies. Our *custom* (precise) DFA implementation shows a positive but non-significant correlation between ages and complexity, which attenuates the contradictory aspect of Hausdorff's study. The *casnet* package was the only one that showed a negative correlation which is in accordance with Hausdorff's results but this package showed significant biases for white noise signals (see previous subsection).

The soft DFA (right part of Fig. 5) showed variable evidential bands widths which highlights variable uncertainty levels among children gaits complexities. The correlation computed between children ages and the pignistic expectation of evidential complexities was positive but moderately significant (p-value =

0.06). The use of soft DFA seems to attenuate the contradiction between DFA implementations thanks to uncertainty modelling of regression residuals.

5 Conclusion and perspective

This paper proposes an evidential extension of the DFA that benefits from statistical uncertainties during the final log-regression of the fluctuations-terms data. To do so, an evidential linear regression model was proposed. This imprecise linear regression model computes evidential slopes and intercepts from the residuals computed from precise univariate data $(x_i, y_i)_{i=1,\dots,n}$ through a geometrical approach.

The first contribution of this paper is to lighten some of the DFA steps by considering, for each term, a small number of windows (10 by default) which are randomly picked in the raw signal and to use all of them in the final log-regression step (*custom* DFA). Three R packages (*nonlinearTseries*, *DFA* and *casnet*) that contains DFA implementations have been compared between themselves and with our *custom* DFA implementation. Experiments on sinusoidal signals show the need for a soft approach to capture the signals degree of fractality and the experiments on simulated noise signals show that *nonlinearTseries* was the only consistent R implementation with DFA theorical background. Our *custom* implementation got similar results to *nonlinearTseries* package. This highlights the uselessness of several DFA steps, enabling DFA computation times decrease. An attempt to replicate some of Hausdorff results highlighted DFA reproducibility issues that were attenuated with our proposed soft DFA.

In future works, the evidential linear regression models proposed in this paper could be extended to the multivariate context by considering n-dimensional extensions of evidential bands. In terms of DFA, the evidential linear regression proposed in this paper could be applied during each fluctuation computation. To do so, an evidential regression model that handle evidential needs to be defined in order to regress the resulting evidential fluctuations. The use of multifractals could help considering a probabilistic fractal complexity which could be compared to our soft DFA one.

References

1. Amini, A., Schwarting, W., Soleimany, A., Rus, D.: Deep evidential regression. In: Larochelle, H., Ranzato, M., Hadsell, R., Balcan, M., Lin, H. (eds.) Advances in Neural Information Processing Systems (NeurIPS'2020). vol. 33, pp. 14927–14937. Curran Associates, Inc. (2020)
2. Arsac, L., Deschodt-Arsac, V.: Detrended fluctuation analysis in a simple spreadsheet as a tool for teaching fractal physiology. Advances in Physiology Education **42**(3), 493–499 (2018)
3. Bryce, R.M., Sprague, K.B.: Revisiting detrended fluctuation analysis. Scientific reports **2**(1), 315 (2012)
4. Cattaneo, M., Andrea, W.: Likelihood-based imprecise regression. International Journal of Approximate Reasoning **53**(8), 1137–1154 (2012), imprecise Probability: Theories and Applications (ISIPTA'11)

5. Delignières, D., Marmelat, V.: Fractal fluctuations and complexity: current debates and future challenges. Critical Reviews™ in Biomedical Engineering **40**(6) (2012)
6. Dempster, A.: New Methods for Reasoning Towards Posterior Distributions Based on Sample Data. The Annals of Mathematical Statistics **37**(2), 355 – 374 (1966)
7. Denoeux, T.: An evidential neural network model for regression based on random fuzzy numbers. Lecture Notes in Computer Science p. 57–66 (2022)
8. Denœux, T.: Quantifying prediction uncertainty in regression using random fuzzy sets: The ennreg model. IEEE Transactions on Fuzzy Systems **31**(10), 3690–3699 (2023)
9. Fernández, E., Jelinek, H.: Use of fractal theory in neuroscience: methods, advantages, and potential problems. Methods **24**(4), 309–321 (2001)
10. Ferraro, M., Coppi, R., González Rodríguez, G., Colubi, A.: A linear regression model for imprecise response. International Journal of Approximate Reasoning **51**(7), 759–770 (2010)
11. Francis, D., Willson, K., Georgiadou, P., Wensel, R., Davies, L., Coats, A., Piepoli, M.: Physiological basis of fractal complexity properties of heart rate variability in man. The Journal of physiology **542**(2), 619–629 (2002)
12. Goldberger, A., Amaral, L., Hausdorff, J., Ivanov, P., Peng, C.K., Stanley, H.: Fractal dynamics in physiology: alterations with disease and aging. Proceedings of the national academy of sciences **99**(suppl_1), 2466–2472 (2002)
13. Hardstone, R., Poil, S.S., Schiavone, G., Jansen, R., Nikulin, V., Mansvelder, H., Linkenkaer-Hansen, K.: Detrended fluctuation analysis: a scale-free view on neuronal oscillations. Frontiers in physiology **3**, 450 (2012)
14. Hausdorff, J., Ashkenazy, Y., Peng, C.K., Ivanov, P., Stanley, H., Goldberger, A.: When human walking becomes random walking: Fractal analysis and modeling of gait rhythm fluctuations. Physica A **302**, 138–47 (01 2002)
15. Hurst, H.: Long-term storage capacity of reservoirs. Transactions of the American society of civil engineers **116**(1), 770–799 (1951)
16. Lennartz, S., Bunde, A.: Distribution of natural trends in long-term correlated records: A scaling approach. Physical Review E **84**(2), 021129 (2011)
17. Mandelbrot, B.: The fractal geometry of nature, vol. 1. WH freeman New York (1982)
18. Peng, C.K., Buldyrev, S., Havlin, S., Simons, M., Stanley, H.E., Goldberger, A.: Mosaic organization of dna nucleotides. Physical review e **49**(2), 1685 (1994)
19. Peng, C.K., Mietus, J., Hausdorff, J., Havlin, S., Stanley, H., Goldberger, A.: Long-range anticorrelations and non-gaussian behavior of the heartbeat. Physical review letters **70**(9), 1343 (1993)
20. Piergiovanni, S., Terrier, P.: Reliability of a novel method to assess correlation structure among consecutive stride intervals using a low-back accelerometer: A test-retest study. Gait & Posture **97**, S154–S155 (2022)
21. Serrurier, M., Prade, H.: Imprecise regression based on possibilistic likelihood. In: Benferhat, S., Grant, J. (eds.) Scalable Uncertainty Management. pp. 447–459. Springer Berlin Heidelberg, Berlin, Heidelberg (2011)
22. Setty, V.A., Sharma, A.: Characterizing detrended fluctuation analysis of multifractional brownian motion. Physica A: Statistical Mechanics and its Applications **419**, 698–706 (2015)
23. Snell, J.: 6. Alternative Methods of Regression. Journal of the Royal Statistical Society Series A **159**(1), 182–183 (January 1996)