# Ranking of Arguments using Social Ranking Choice

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**Abstract.** In argumentation theory, semantics defined by Dung evaluate subsets of arguments by classifying each into two categories: accepted or rejected. This makes some applications (like online debate) more complex since many accepted arguments can be returned without any insight into the strength of each argument. Conversely to extension-based semantics, ranking-based semantics allow us to determine the strength of acceptability of each argument. However, this approach does not evaluate sets of arguments but each argument individually. In this paper, our goal is to classify the arguments more precisely than just accepting or rejecting them and, therefore, to find a total pre-order of arguments. For this purpose, we will present a method to, first, rank subsets of arguments using extension-based semantics and, then, apply power indices of social choice to this ranking to find a pre-order of arguments. Our approach has the advantage of combining extension-based semantics and lexicographic social ranking. Indeed, given two arguments, it allows us to state which one is more plausible than the other and if they are jointly acceptable or not.

**Keywords:** Abstract argumentation, Ranking, Semantics

## 1 Introduction

Abstract argumentation is a discipline that studies how arguments coexist. It is based on philosophical and informal theories of argumentation. Arguments are considered as nodes in a graph, abstracting away from their actual content. The discipline was initiated by Dung [11], where an argumentation framework is depicted as a graph in which arguments are nodes and an argument $a$ attacking an argument $b$ is represented by an arrow from node $a$ to node $b$. The field has since evolved, particularly to meet the demands arising from applications in computer science. This evolution involves adding supporting relationships, strength of arguments [5], preference among arguments [10], and more. It provides a systematic approach to dealing with conflicting information and uncertain reasoning by modeling the process of argumentation and debate. In the context of AI, this allows rational decisions based on the information available. These decisions are rational because they employ different semantics which have been defined, starting with Dung [11]. A semantic can be seen as capturing a particular notion of rationality, ranging from less demanding to more demanding. They allow us to determine whether a set of arguments can be jointly accepted or not. So, arguments were classified into two categories: accepted and rejected by using extension-based semantics defined by Dung [11]. However, an argumentation framework can count many arguments, requiring a more precise ranking than just two categories. Instead of binary classifying arguments as accepted or rejected, the problem of determining the strength of each argument has recently received considerable interest. The aim is to associate every argumentation framework with an ordering over the arguments (usually a total pre-order) according to their degree of acceptability using the various ranking-based semantics proposed in the literature [6,7,5,10,4,17,18,9]. This approach evaluates each argument individually, but does not allow to state that a set of arguments is jointly acceptable.

To overcome these problems, we propose an approach combining extension-based semantics and ranking-based semantics. The method follows two steps: The first step establishes a ranking over sets of arguments based on the inclusion relation of extension-based semantics, including stable, preferred, complete, admissible and conflict-free. These semantics indicate whether a set of arguments is acceptable and whether a set of arguments is more plausible than another set. The second step uses ranking-based semantics, particularly Power Index-based semantics [7], which employs Banzhaf and Shapley power indices to refine the ranking established in the first step. Power indices are tools that will allow measuring the influence of an argument based on other arguments. They are beneficial in various fields, such as multicriteria analysis, cooperative game theory [13] and machine learning [16]. Our approach allows us to rank arguments and determine if a set of arguments are jointly accepted. Namely, it will enable going from a ranking over sets of arguments to a ranking over arguments.

The paper is organized as follows: In the next section, we will recall what an argumentation system is and the various semantics that we will use. Section 3 describes and gives the proposed method for ranking arguments following two main steps. Section 4 discusses some of the properties proposed in the literature and introduces new ones. Section 5 gives some related works, and finally, we conclude the paper.

## 2 Preliminaries

In this section we present Dung's argumentation framework [11] and the main semantics used in the literature.

**Definition 1.** *[11] An argumentation framework (AF) is a pair $AF=\langle \mathcal{A}, \mathcal{R} \rangle$ with $\mathcal{A}$ a finite set of arguments and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ an attack relation between arguments. An argument $a \in \mathcal{A}$ attacks an argument $b \in \mathcal{A}$, if $(a, b) \in \mathcal{R}$. An argument $c \in \mathcal{A}$ defends $b$ against an attacker $a$ if $c$ attacks $a$. We denote by Att($a$) the set of all attackers of $a$ in $\mathcal{A}$ (i.e. Att($a$)=$\{b \in \mathcal{A} | (b, a) \in \mathcal{R}\}$) and by Def($a$) the set of all defenders of $a$.*

Argumentation frameworks can be represented by a directed graph, where nodes are arguments, and edges are attack relations between two arguments. Several semantics have been defined to select a set of accepted arguments; such sets are called **extensions**. We recall the definitions of the main semantics on which our method is based.

**Definition 2.** *[11] Given an argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$, a set of arguments $\mathcal{S} \subseteq \mathcal{A}$ is **conflict free** in $AF$ if $\forall a, b \in \mathcal{S}$, $(a, b) \notin \mathcal{R}$.*

*This extension facilitates the presentation of a set of arguments without any inter-argument attacks. Naturally, an AF can manifest multiple distinct conflict-free extensions. Given the context of the problem, there may be a desire to impose supplementary constraints on the arguments deemed acceptable. We present below some of these constraints, which are frequently employed in argumentation theory:*

*A conflict free set $\mathcal{S}$ is **admissible** if it defends all its arguments against each of their attackers: i.e., for every attacker of arguments in $\mathcal{S}$, there exists an argument in $\mathcal{S}$ that defends it. An admissible set $\mathcal{S}$ is:*

- *a **complete extension** if each argument defended by $\mathcal{S}$ belongs to $\mathcal{S}$.*
- *a **preferred extension** if it is a $\subseteq$-maximal set admissible of $AF$.*
- *a **stable extension** if it attacks each argument in $\mathcal{A} \backslash \mathcal{S}$.*

*We denote by $\epsilon_\sigma$ the set of extensions of $AF$ for the semantics $\sigma \in$ {cf, ad(missible), co(mplete), pr(eferred), st(able)}.*

*Example 1.* In Figure 1, we have an argumentation framework $AF$ composed of $\mathcal{A} = \{a, b, c, d, e\}$ and $\mathcal{R} = \{(a, b), (b, c), (e, b), (b, d), (d, b), (d, e), (e, d)\}$.

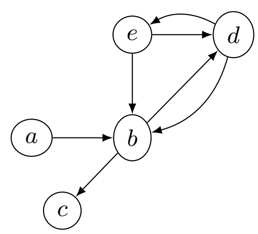


**Fig. 1.** An example of a Dung's Argumentation Framework.

The sets of extensions for the conflict-free, admissible, complete, preferred and stable semantics are respectively: $\epsilon_{\text{cf}}$ = {∅, {a}, {b}, {c}, {d}, {e}, {a, c}, {a, d}, {a, e}, {c, d}, {c, e}, {a, c, d}, {a, c, e}}, $\epsilon_{\text{ad}}$ = {∅, {a}, {d}, {e}, {a, c}, {a, d}, {a, e}, {c, d}, {c, e}, {a, c, d}, {a, c, e}}, $\epsilon_{\text{co}}$ = {{a, c}, {a, c, d}, {a, c, e}} and $\epsilon_{\text{pr}} = \epsilon_{\text{st}}$ = {{a, c, d}, {a, c, e}}.}

As we explained in the introduction, our approach will be grounded in the concept of power indices derived from cooperative game theory. To be more precise, we will use the Social Ranking Theory which makes use of an ordinal version of power indices, as it is defined by Moretti et al. [2]. The objective of Social Ranking is as follows: when presented with an order on coalitions (groups of individuals), the aim is to establish an order on individual entities. In our framework, arguments assume the role of individuals, with coalitions representing subsets of arguments. Consequently, our task is to delineate a procedure for establishing an order on arguments based on an existing order on subsets of arguments. To accomplish this, it is essential to initially define an ordering mechanism for subsets of arguments. We denote the order on coalitions by $\sqsupseteq$ as in the following and we suppose that it is a pre-order (transitive and reflexive).

**Definition 3.** *Let $AF$=$\langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework. A ranking $\sqsupseteq$ on the coalitions, is a pre-order $\sqsupseteq$ on $2^{\mathcal{A}}$. A coalition of arguments $\mathcal{S}$ is at least as acceptable as a coalition of arguments $\mathcal{X}$ if $\mathcal{S} \sqsupseteq \mathcal{X}$. $\mathcal{S} \sim \mathcal{X}$ is a shortcut for $\mathcal{S} \sqsupseteq \mathcal{X}$ and $\mathcal{X} \sqsupseteq \mathcal{S}$, and $\mathcal{S} \sqsupset \mathcal{X}$ is a shortcut for $\mathcal{S} \sqsupseteq \mathcal{X}$ and $\mathcal{X} \not\sqsupseteq \mathcal{S}$.*

Using this relation on the coalitions, our aim is to define an order on the arguments, that we call a *social ranking* on arguments.

**Definition 4.** *A social ranking $\succeq$ on the arguments, is a pre-order $\succeq$ on $\mathcal{A}$. An argument $a$ is at least as acceptable as an argument $b$ if $a \succeq b$. $a \sim b$ is a shortcut for $a \succeq b$ and $b \succeq a$, and $a \succ b$ is a shortcut for $a \succeq b$ and $b \not\succeq a$.*

## 3 Ranking of Arguments

Our objective is to associate every argumentation framework with a ranking of arguments (social ranking). The method comprises two steps: 1) Establishing a first ranking over sets of arguments on the basis of extension-based semantics [11]. The ranking returned at this stage considers the inclusion relation between semantics (see Figure 2). 2) Refining the ranking established in the first step using Power Index-based semantic [7], which employs Banzhaf and Shapley power indices. Power indices are tools that will allow measuring the influence of an argument based on other arguments. They are beneficial in various fields, such as multicriteria analysis and cooperative game theory. To the best of our knowledge, our approach is the first that combines these indices with an input of an ordinal ranking.

### 3.1 Rankings over sets of arguments by considering the inclusion relation between semantics

An inclusion relationship exists among the different semantics [11]. Any stable extension is a preferred extension; any preferred extension is a complete extension; any complete extension is an admissible extension, and any admissible extension is a conflict-free extension (see Figure 2).

From an argumentation framework, we will retrieve the extensions: conflict-free, admissible, complete, preferred, and stable. Then, we exploit the inclusion relationship to classify these extensions based on the semantics to which they belong. We will create a ranking by placing stable sets in the first position; then in the second position, the sets obtained with the semantic {preferred}\{stable}. Next, we will place the sets {complete}\{stable, preferred}, followed by the sets {admissible}\{stable, preferred, complete}, and finally the {conflict-free}\{stable, preferred, complete, admissible}. All the remaining sets will be placed in the last position as they do not belong to any semantics and are, therefore, unacceptable. In the end, we will have a ranking with six equivalence classes with possible gaps that we represent by () with abuse of notation: $\Sigma_1 \sqsupseteq \Sigma_2 \sqsupseteq \Sigma_3 \sqsupseteq \Sigma_4 \sqsupseteq \Sigma_5 \sqsupseteq \Sigma_6$, with in $\Sigma_1$ the sets in $\epsilon_{\text{st}}$, in $\Sigma_2$ the sets in $\epsilon_{\text{pr}}$, in $\Sigma_3$ the sets in $\epsilon_{\text{co}}$, in $\Sigma_4$ the sets in $\epsilon_{\text{ad}}$, in $\Sigma_5$ the sets in $\epsilon_{\text{cf}}$ and in $\Sigma_6$ the rest.

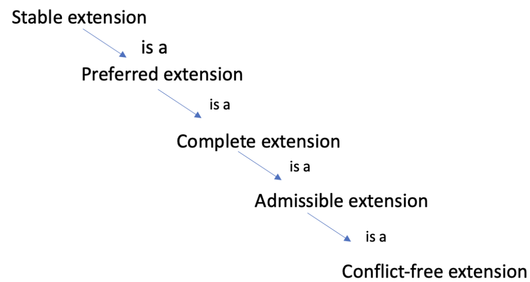


**Fig. 2.** An overview of the inclusion relation of different semantics.

*Remark 1.* There is another well-known extension we do not use, that is, grounded. Any grounded extension is a complete extension. Therefore, by using this semantic, we would have fewer equivalence classes. Since our goal is to achieve a more detailed ranking, we have focused on the semantics depicted in Figure 2.

*Example 2.* The ranking obtained by using the inclusion relationship between the semantics for the $AF$ of Figure 1 is as follows: $(\{a,c,e\} \sim \{a,c,d\}) \sqsupseteq () \sqsupseteq (\{a,c\}) \sqsupseteq (\emptyset \sim \{a\} \sim \{e\} \sim \{d\} \sim \{a,e\} \sim \{a,d\} \sim \{c,d\} \sim \{c,e\}) \sqsupseteq (\{b\} \sim \{c\}) \sqsupseteq rest.$

*Remark 2.* With abuse of notation, in the rest of the paper, we will denote the coalition "abc" instead of {a, b, c} for better readability. The symbol $\sim$ represents that the coalitions are in the same extension.

This method is justified by inclusion. By placing the most restrictive extensions, i.e., those that appear in multiple semantics, at the top of the ranking, we consider the most stringent criteria. This approach is also justified by the notions of skeptically and credulously accepted arguments. We refer to an argument as "skeptically accepted" if it is present in all the extensions of a chosen semantics. If found in at least one extension, it is considered "credulously accepted." A sceptically accepted argument will rank higher than a credulously accepted one. We can draw a parallel with our method and state that coalitions in the stable extension are skeptically accepted and thus ranked higher because they appear in all the extensions. Therefore, we justify our ranking by prioritizing those that appear in more extensions, as they are considered more widely accepted.

As we can observe, the result of this step concerns a ranking over sets of arguments. The second step of our method consists in considering the ranking obtained by using the inclusion relation between semantics, and some well-known ordinal power indices [2] to establish a social ranking of arguments. In the existing literature, various ordinal power indices have been put forth, each exhibiting preferences for specific aspects, such as: we can give greater importance to arguments that enable a coalition to be accepted, i.e., their marginal contribution (as in the Banzhaf ordinal index, see for instance [15]); or we can prioritize arguments that appear at the top of the ranking and thus in numerous extensions (as in the Lexicographic index, see for instance [1]) or we can compare arguments pairwise in a Ceteris Paribus way, seeing each coalition as a potential voter in favor or not for one of the individual (using the CP majority index, see for instance [12]) like in Social Choice Theory when we use the notion of Condorcet winner.

### 3.2 Using lexicographic social ranking rules for ranking of arguments

We choose the lexicographic index [1] to construct a ranking over arguments and to expose some properties. We chose this index to highlight the arguments at the top of the list, as they appear in more possible extensions according to our ranking of coalitions based on inclusion relationships. Moreover, this social ranking method is clearly defined, with its axiomatisation documented (refer to [1]). Additionally, it demonstrates commendable behavior in relation to manipulation strategies (see [2]). We start by presenting this index. We denote by $i_k$ the number of sets in the equivalence class $\Sigma_k$ containing argument $i$:

$$i_k = |\{\mathcal{S} \in \Sigma_k : i \in \mathcal{S}\}| \tag{1}$$

for $k = 1 \ldots 6$. If an equivalence class $k$ is empty, then $i_k = 0$ for all arguments $i$. Now, let $\theta(i)$ be the 6-dimensional vector $\theta(i) = (i_1, ..., i_6)$ associated to the ranking of coalitions. Consider the lexicographic order $i \geq_L j$ if either $i = j$ or there exists $t : i_r = j_r, r = 1, ..., t$-1, and $i_t > j_t$.

**Definition 5.** *The lexicographic excellence (lexcel) [1] relation is the binary relation* $\succeq$ *such that for all* $i, j \in \mathcal{A}$*:* $i \succeq j \iff \theta(i) \geq_L \theta(j)$.

*Example 3.* Let us apply lexcel to find the social ranking of arguments on the ranking of coalitions of Example 2 based on Figure 1. First, we remind that we have: $\Sigma_1 = (\{a, c, e\}, \{a, c, d\})$, $\Sigma_3 = (\{a, c\})$, $\Sigma_4 = (\emptyset \sim \{a\} \sim \{e\} \sim \{d\} \sim \{a, e\} \sim \{a, d\} \sim \{c, d\} \sim \{c, e\})$,$\Sigma_5 = (\{b\} \sim \{c\}) \sqsupseteq rest$.

Then, we have: $\theta(a) = (2, 0, 1, 3, 0, .)$[3], $\theta(b) = (0, 0, 0, 0, 1, .)$, $\theta(c) = (2, 0, 1, 2, 1, .)$, $\theta(d) = (1, 0, 0, 3, 0, .)$ and $\theta(e) = (1, 0, 0, 3, 0, .)$.

At the end, we obtain a social ranking of arguments: $a \succeq c \succeq e \sim d \succeq b$ [4]

*Remark 3.* The inclusion of the empty set as an extension is acknowledged; nonetheless, its presence or absence does not influence the resultant order when employing the lexcel method. It is important to highlight that this observation may not hold universally across all social ranking methods; for instance, the ordinal Banzhaf index may exhibit sensitivity to the presence or absence of the empty set.

In evaluating the outcomes of our method, a comparative analysis will be conducted with another approach: the Power Index (PI)-Based Semantics introduced previously in the literature ([7]). This comparison is pertinent as the PI-Based Semantics also leverages power indices and incorporates coalitions, aligning with the methodology outlined in our work. We present the following PI-based semantic, which is based on Banzhaf scoring:

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[3] Because $a$ appears two times in $\Sigma_1$, zero time in $\Sigma_2$, one time in $\Sigma_3$, etc.

[4] For instance in order to say that $a \succeq c$, we compare the two vectors $\theta(a)$ and $\theta(c)$ in a lexicographic way (2=2, 0=0, 1=1 but 3>2).

The **Banzhaf scoring** of a characteristic function $v_\sigma$ on $2^{|\mathcal{A}|}$, attributes to argument $i$ the score:

$$\pi_i^{\text{Ban}}(v_\sigma) \;=\; \sum_{S \in 2^N:\, i \notin S} \frac{1}{2^{n-1}} \left(v_\sigma(S \cup \{i\}) - v_\sigma(S)\right). \tag{2}$$

The **characteristic function** $v_\sigma$ for semantic $\sigma$, with a set of argument $\mathcal{S} \subseteq \mathcal{A}$ is:

$$v_\sigma(\mathcal{S}) \;=\; \begin{cases} 1, & \text{if } \mathcal{S} \in \epsilon_\sigma; \\ 0, & \text{if otherwise .} \end{cases}$$

One of the main differences between our approach and the PI-based one is that the PI-based one chooses first a semantic and then makes a ranking while our approach makes use of 5 different semantics, as illustrated in the next example.

*Example 4.* By applying the PI-based semantic with the Banzhaf scoring on the stable semantic on the argumentation framework in Figure 3, we find: $a \sim b \sim c \sim d$. However, if we apply our method, we get:

- By using the inclusion relationship between semantics, we obtain the following ranking over sets of arguments: $(ad \sim bc) \sqsupseteq () \sqsupseteq () \sqsupseteq (\emptyset \sim d \sim a \sim b) \sqsupseteq (c \sim cd) \sqsupseteq rest$.
- Applying lexcel on the obtained ranking, we have: $d \succeq a \sim b \succeq c$.

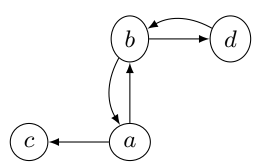


**Fig. 3.** An AF example.

This example illustrates that our approach, leveraging more information than the PI-based method thanks to the use of many semantics and their relation, excels in achieving a more refined order of the arguments.

# 4 Properties

A ranking can be characterised by specific properties that consider how couples of arguments in an AF are evaluated for establishing their position. We provide a list of the properties inspired by [7] and [3] that our social ranking satisfies. We start by introducing two notions which are necessary to define our properties.

**Definition 6.** *An isomorphism f between two argumentation frameworks* $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ *and* $AF' = \langle \mathcal{A}', \mathcal{R}' \rangle$ *is a bijective function f:* $\mathcal{A} \rightarrow \mathcal{A}'$ *such that* $\forall a, b \in \mathcal{A}$*,* $(a, b) \in \mathcal{R}$ *if and only if* $(f(a), f(b)) \in \mathcal{R}'$*.*

**Definition 7.** *A connected component in a graph is a subset of nodes of the graph in which every pair of nodes is connected by a path (a sequence of nodes and edges) within that component.*

**Abstraction** [7]: The ranking on arguments $\mathcal{A}$ is defined only on the basis of the attacks between arguments, that is, it is preserved over isomorphisms of the framework.
**Independence** [7]: The ranking between two arguments $a$ and $b$ should be independent of any argument that is neither connected to $a$ nor to $b$.
**Void-precedence** [7]: A non-attacked argument is ranked strictly higher than any attacked argument.
**Self-contradiction** [7]: A self-attacking argument is ranked strictly lower than any non self-attacking argument.
**Totality** [7]: All pairs of arguments can be compared.

**Proposition 1.** *The social ranking of arguments obtained by applying our method satisfies: abstraction, independence, void-precedence, self-contradiction and totality.*

*Proof.*

- Abstraction: the ranking of arguments is defined only on the basis of the attacks between arguments, that is it is preserved over isomorphisms of the framework.
- Independence: the method we propose computes the ranking starting from the sets of extensions of each semantic. Since the status (accepted or rejected) of each argument is determined by the other arguments in the same connected component, also the ranking between every pair of arguments $a$ and $b$ is independent of any other argument outside the connected component of $a$ and $b$.
- Void-precedence: there are two cases of attacked argument. First, let us take an argument $b$ which is attacked, but defended by another (not attacked) argument $a$. Then $b$ will be in (a possibly stable extension) a preferred extension and complete all the time with argument $a$, so $a$ will have at least the same number for the first 3 components of the vector $\theta(a)$ of lexcel. However, contrary to argument $a$, the argument $b$ alone will not be in the admissible extension since it is attacked while $a$ will be, so lexcel will rank $a$ higher.
  Let us now turn to the second case, which is the case if the argument is not defended, then it will be at the bottom of the ranking in conflict-free or in the rest. So a non-attacked argument is ranked strictly higher than any attacked argument.
- Self-contradiction: self-attacking argument cannot be in any extension, so it will be at the end of the coalition ranking and therefore at the end of the social ranking of arguments.

**Cardinality Precedence** [7]: The greater the number of direct attackers for an argument, the weaker the rank of this argument.
**Defence Precedence** [7]: For two arguments with the same number of direct attackers, a defended argument is ranked strictly higher than a non-defended argument.

**Non-attacked Equivalence** [7]: All the non-attacked arguments have the same rank.
**Maximality** [3]: If an argument is not attacked, then the argument is first in the ranking.
**Symmetry** [3]: $\forall a, b \in \mathcal{A}$, if $a$ and $b$ have the same attackers , then $a$ and $b$ are at the same rank.

**Definition 8.** *The social ranking saying $b \succeq a$ is not coherent with the AF, if $a$ attacks $b$, otherwise it is coherent.*

**Neutrality** [3]: $\forall a, b \in \mathcal{A}$, if $Att(b) = Att(a) \bigcup \{x\}$ such that $x$ is not coherent with $b$, then $a$ and $b$ have the same rank.
**Monotony** [3]: $\forall a, b \in \mathcal{A}$, if $Att(a) \subseteq Att(b)$, then $a$ is ranked no lower than $b$.
**Strict Monotony**: $\forall a, b \in \mathcal{A}$, if $Att(a) \subseteq Att(b)$ and $Def(b) \subseteq Def(a)$ then a is ranked no lower than b.
**Quality Precedence** [7]: An argument $a$ should be ranked strictly higher than an argument $b$, if at least one attacker of $b$ is ranked strictly higher than any attacker of $a$.

**Proposition 2.** *The social ranking of arguments obtained by applying our method does not satisfy: cardinality precedence, defence precedence, non-attacked equivalence, symmetry, maximality, neutrality, monotony, strict monotony and quality precedence.*

*Proof.*

- Figure 4 is a counter-example for $cardinality\ precedence$ because using our method, the ranking over sets of arguments is $(ace) \sqsupseteq () \sqsupseteq () \sqsupseteq (ac \sim ae \sim ce \sim a \sim c) \sqsupseteq b \sim d....$
  By lexcel on this ranking, we have that $e \succeq b$ and $e \succeq d$. However, $e$ has a greater number of direct attackers than $b$ and $d$.

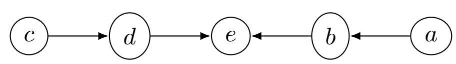


**Fig. 4.** Counter-example.

- Figure 5 is a counter-example for $defence\ precedence$ because $e$ and $c$ have each one attacker, $e$ has no defender, $c$ has one defender: $a$, but $e \succeq c$.
- Figure 5 is a counter-example for $non\text{-}attacked\ equivalence$ because $f$ and $d$ are non attacked but $f \succeq d$.
- $Maximality$ is not satisfied since we do not have non-attacked equivalence (Figure 5 is a counter-example).
- Figure 6 is a counter-example for $symmetry$ because $c$ and $b$ have the same attackers: $a$ but $b \succeq c$.
- $Neutrality$ is not satisfied since we do not have symmetry.
- Figure 3 is a counter-example for $monotony$ because $Att(c) = a \subseteq Att(b) = a, d$, but $b \succeq c$.
- Figure 3 is a counter-example for $strict\ monotony$ because $Att(c) = a \subseteq Att(b) = a, d$ and $Def(b) = b \subseteq Def(c) = b$, but $b \succeq c$.
- Figure 7 is a counter-example for $quality\ precedence$ because by lexcel we find the social ranking: $a \succeq d \succeq f \succeq c \succeq b \succeq e$. $b$ is attacked by $a$ with $a$ ranked strictly higher than any argument and so any attacker of $e$. But we have $b \succeq e$.

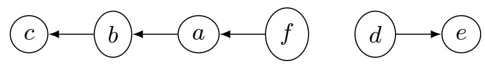


**Fig. 5.** Counter-Example. The ranking of coalitions: $(bdf) \sqsupseteq () \sqsupseteq () \sqsupseteq (\emptyset \sim f \sim d \sim df \sim bf) \sqsupseteq (\emptyset \sim e \sim c \sim ce \sim fe \sim fc \sim fce \sim cd \sim fcd \sim b \sim be \sim fbe \sim bd \sim a \sim ae \sim ac) \sqsupseteq rest$.

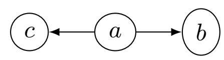


**Fig. 6.** Counter-example. The ranking of coalitions: $(bc \sim a) \sqsupseteq () \sqsupseteq () \sqsupseteq (\emptyset \sim b) \sqsupseteq c \sqsupseteq rest$.

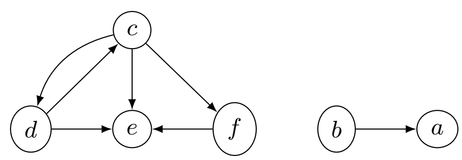


**Fig. 7.** Counter-example. The ranking of coalitions: $(adf \sim ac) \sqsupseteq () \sqsupseteq (a) \sqsupseteq (\emptyset \sim d \sim df \sim ad \sim c) \sqsupseteq (f \sim b \sim bf \sim bd \sim bdf \sim af \sim e \sim be \sim ae \sim bc) \sqsupseteq rest$.

## 5 Related Work

Instead of just returning a set of extensions of an argumentation framework when we use extension-based semantics, several approaches have been developed to give a more detailed evaluation of arguments. The so-called ranking-based semantics [6,7,5,10,4,17,18,9] aim to rank-order arguments according to their degree of acceptability using ranking functions (e.g., h-categoriser function [6]). The main problem is that they do not allow to determine whether a set of arguments is jointly acceptable. For example, even if two arguments have high degrees of acceptability, they may not be jointly acceptable since they conflict.

Some authors have recently considered this question to overcome this problem. In [14], the authors defined extension-ranking semantics, which is a generalisation of extension-based semantics to determine whether a set of arguments is jointly acceptable and also whether a set is more plausible than another set. For example, given two sets of arguments $E$ and $E'$, $E$ is more plausible than $E'$ regarding conflict-freeness if $E$ has strictly less conflicts than $E'$ (w.r.t. set inclusion). Hence, an argument $a$ is at least as plausible as $b$ if $a$ is contained in $E$. In [19], the author discussed the relationship between ranking-based and extension-ranking semantics and showed that these two semantics can be transformed into each other. Namely, going from a ranking over arguments to a ranking over sets of arguments and from a ranking over sets of arguments to a ranking over arguments. In the same direction, the idea of combining ranking-based semantics and extension-based semantics is studied in some propositions. Among these approaches, the one proposed in [8] uses extension-based semantics to improve ranking-based semantics and vice versa. For example, they exploit the ranking of arguments returned by a ranking function to select the best extensions for a given semantic. The work proposed in [14] also allows rank-order the set of arguments based on the semantics defined for ranking extensions. Other works following this idea can be found in [20,19].

The ranking we presented in this paper is considered a social ranking problem since it is based on social ranking techniques used in the computation of social choice [15,12]. The approach proposes to go from a ranking over sets of arguments to a ranking over arguments. In our context, the ranking over sets of arguments is obtained from the inclusion relationship between conflict-free, admissible, complete, preferred and stable semantics. The methods cited here for ranking single arguments or over sets of arguments are based on a single semantic. To the best of our knowledge, our method is unique in that it considers several semantics by exploiting the inclusion relation between them. This allows for a more detailed evaluation of arguments.

## 6 Conclusion

In this paper, we have presented a method to find a ranking of coalitions and a ranking of arguments. Unlike other rankings based on semantics, we first defined the ranking of coalitions and then used power indices to obtain a social ranking of arguments. Additionally, instead of relying on a single semantic, we used some of Dung's main semantics to achieve higher ranking precision and we analysed different properties. For future work, we plan to study the last equivalence class that includes all the coalitions not accepted by any semantics. One possible line of investigation is to draw inspiration from the proposal in [14] to rank these coalitions. Furthermore, to deepen the comparison of our method with the existing ones, we aim to perform graph simulations and observe the results obtained for the different techniques.

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