The hybrid method of inference system based on experts' rules and machine learning with an uncertainty aspect

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Abstract. Rule-based approximate reasoning systems are an important decision-making tool in many application problems. The use of expert knowledge or machine learning techniques to create rules does not exhaust the problems of representing data and decision dependencies, therefore we propose a hybrid/mixed technique for creating a set of rules while effectively modeling uncertainty through interval-valued fuzzy representation in the problem of detecting falls of elderly people. The obtained prediction confirms the correctness of the choice of diagnostic methodology.

1 Introduction and motivation

In many decision-making issues, especially in everyday medical practice, we meet a problem of uncertain data thus the reasoning for choosing the research direction was the manipulation of uncertainty in classification problems. Uncertainty, also understood here as imprecision, is effectively represented using interval fuzzy sets, which is confirmed by numerous application examples. There the beginning and end of the interval mean the limit values between which there is one desired value (epistemic approach - the source of uncertainty is the lack of precise knowledge) [9]. In particular, uncertainty may be of an objective nature (caused by the complexity or nature of the phenomenon), subjective (caused by the personal opinion, interpretation, or lack of conviction of the decision-maker), or caused by the low quality of information. Additionally, by their very nature, medical descriptions are often imprecise and ambiguous, e.g. they depend on the medical equipment used or the doctor's interpretation. This state of affairs requires the use of non-classical methods of data modeling and inference, i.e. methods that take into account imprecision. Despite much work in this field (e.g. [4, 2]), there are still no effective methods to handle this type of imprecision in medicine or industry. Information and uncertainty are intimately connected and the most fundamental aspect of this connection is that uncertainty involved in any situation is a result of some information deficiency. On the other hand, information may be imprecise, fragmentary, not fully reliable, vague, incomplete, or even contradictory. Assumably, various information deficiencies may result in different types of uncertainty. As a result of the need for a convenient description of non-random forms of uncertainty, the theory of fuzzy sets appeared. Fuzzy set extensions have been very useful, in particular interval-valued fuzzy sets (IVFSs), which are the basis of this contribution. In this work, we focus on the application of IVFS in approximate inference in posture detection, a fall detection system for elderly people. There the set of rules and their origins play a key role. Their origin, regardless of the source: expert, or created using the machine learning method, also carries uncertainty regarding the source of data, measurements, or human assessments. Therefore, in this work, we propose an approach that takes into account both sources/methods of creating a set of rules used in the generalized approximate inference model, i.e. a hybrid method of generating a set of rules in the interval-fuzzy inference model. The practical aspect, i.e. the detection of falls in older people, is an important and necessary issues to support in today's highly developed societies. Therefore, we consider the use of interval calculus with a new technique to create a set of rules, to represent uncertainty in two aspects:

1. representation of imprecise data;
2. proposing a hybrid method to create rules of inference system and compare this with based on experts and decision trees methods.

A diagram illustrating the decision-making process presented in this "Hybrid inference system (HIS)" work is visualized in Figure 1.

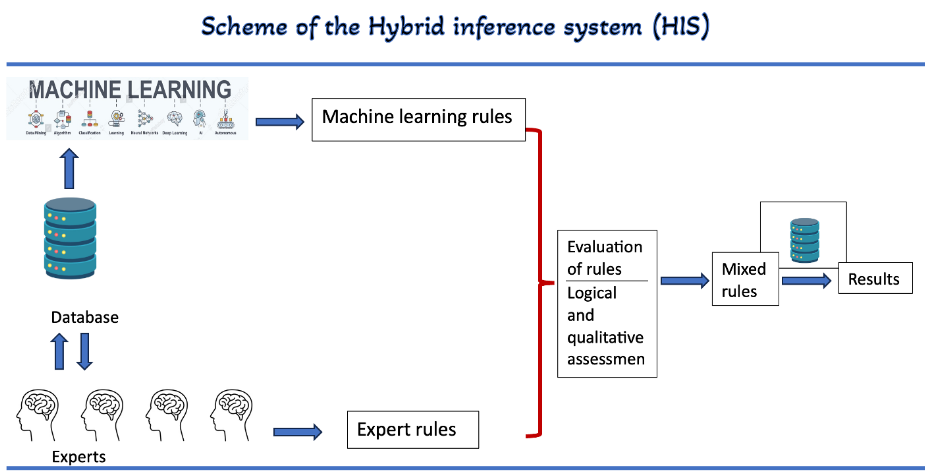

Fig. 1. HIS

2 Background

Artificial intelligence techniques provide various methods to create rules. It becomes very helpful when expert knowledge is out of reach, and one cannot use it to create expert rules, or in a case when expert rules are too numerous. Lately lot of articles depict different methods to create a set of rules. For example in the recent work of Huang, Gao, Li, and Zhang, Reinforcement Learning and Graph Neural Networks, were used to create a set of rules [13]. The results of this study show that this approach outperforms other approaches commonly used in such a problem. In the work of Dixit and Jain rule generation was obtained by an intuitionistic fuzzy clustering technique that was implemented with various window sizes [8]. This helped to significantly reduce the mean square error. Research was done on different databases. On the other hand, a lot of researchers work on not only generating rules but also optimizing them. For instance Gao and Bi [11] after obtaining the rule set, calculate the weights of each rule and their overall normalized influence. The final step in the work was to remove all redundant rules. These procedures lead to gaining similar results as other machine learning approaches while having smaller computational complexity. In the article written by Cintra, Monard, Helposa, and Camargo [7] authors presented genetic algorithms-based methods to generate rules, which were SLAVE, FCA-Based and MPLCS, and decision tree-based methods, such as: C4.5, PART and FUZZYDT. Results show that among all methods best results bring the C4.5 algorithm.

3 Uncertainty, fundamentals of interval-valued theory

In meanwhile, with the introduction of fuzzy sets by Zadeh [25], many approaches and theories to study and model uncertainty have been suggested. Particularly, interval-valued fuzzy sets [20, 24] are an effective tool for uncertainty modeling in a lot of real problems.

3.1 Interval-valued fuzzy setting

Crucial in our approach will be the expression of uncertainty by intervals. So a family of intervals belonging to the unit interval will be denoted by $L^I = \{[\underline{p}, \overline{p}] : \underline{p}, \overline{p} \in [0,1],\ \underline{p} \leq \overline{p}\}$. We also recall the definition of **interval-valued fuzzy set** (IVFS) ([24], [20], [23], [12]), i.e. S in X as a mapping $S : X \to L^I$ such that for every one $x \in X$, $X \neq \emptyset$, and $S(x) = [\underline{S}(x), \overline{S}(x)]$ means the degree of membership of an element x into S. The set of all IVFSs in X we mark by IVFS(X). We assume, concerning the application aspect, that $X = \{x_1, \ldots, x_n\}$ is a finite set. In IVFSs, the membership of an element x is not exactly indicated. We have only specified an upper and lower bound of the possible membership. For any fixed $x \in X$ we assume $S(x) = [\underline{S}(x), \overline{S}(x)] = [\underline{s}, \overline{s}]$.

In L^I is often used the best-known partial order $[\underline{s}, \overline{s}] \leq_2 [\underline{t}, \overline{t}] \Leftrightarrow \underline{s} \leq \underline{t}$ *and* $\overline{s} \leq \overline{t}$. But in real-life problems, we must often be capable of comparing data represented by intervals, and then we meet problems with incomparability. Then we may omit this by extending the partial order $\leq_2$ to a linear, called admissible, $\leq_{Adm}$ ([5, 26]).

3.2 Aggregation process

The adaptation of uncertain operators and uncertain data is very essential for the description of reality in a sufficient mathematical model. Particularly, the aggregation function on L^I is a very relevant and needed concept in many aspects of applications (e.g., [10, 18] or [3]), where it is important for gathering high-quality, summary information of data to create accurate results in given decision-making problems. Thus, aggregation is the process of representing data after the merger. For the considered input data in the representation of interval-valued fuzzy sets, we can define aggregations to adequate order $\leq_2$ or $\leq_{Adm}$.

Definition 1 ([26, 3, 15]). *Let $n \in \mathbf{N}$, $n \geq 2$. An operation $\mathcal{A} : (L^I)^n \to L^I$ is called an interval-valued (I-V) aggregation function if it is increasing with regard to the order $\leq$ (partial or linear), i.e.*

$$\forall x_i, y_i \in L^I \quad x_i \leq y_i \Rightarrow \mathcal{A}(x_1, ..., x_n) \leq \mathcal{A}(y_1, ..., y_n) \tag{1}$$

$$\textit{and} \;\; \mathcal{A}(\underbrace{[0,0], ..., [0,0]}_{n\times}) = [0,0], \;\; \mathcal{A}(\underbrace{[1,1], ..., [1,1]}_{n\times}) = [1,1].$$

Beginning from 1988 (Yager) has been known, often discussed, and applied to many practical concepts, OWA operators. The concept of OWA may be extended to the interval-valued setting which is the next generalization of arithmetic mean, aggregation, and additional also with different orders. What is crucial, OWA operators are a particular case of more general aggregation functions called Choquet integrals. In [6] authors used to extend the definition of OWA operators to the class of linear/admissible orders on L^I for interval-valued fuzzy casing as follows:

Definition 2 ([6]). *Let $\leq$ be an admissible order on L^I, and $w = (w_1, \ldots, w_n) \in [0,1]^n$, with $w_1 + \cdots + w_n = 1$. The interval-valued ordered weighted averaging (OWA) operator (IVOWA) associated with $\leq$ and w is a mapping $IVOWA_{\leq,w} : (L^I)^n \to L^I$, given by*

$$IVOWA_{\leq,w}([\underline{x}_1, \overline{x}_1], \ldots, [\underline{x}_n, \overline{x}_n]) = \sum_{i=1}^{n} w_i \cdot [\underline{x}_{(i)}, \overline{x}_{(i)}],$$

where $[\underline{x}_{(i)}, \overline{x}_{(i)}]$, $i = 1, \ldots, n$, denotes the i-th greatest of the inputs with respect to the order $\leq$ and $w \cdot [\underline{x}, \overline{x}] = [w\underline{x}, w\overline{x}]$, $[\underline{x}_1, \overline{x}_1] + [\underline{x}_2, \overline{x}_2] = [\underline{x}_1 + \underline{x}_2, \overline{x}_1 + \overline{x}_2]$.

Insomuch as $IVOWA_{\leq,w}$ is not an aggregation function with respect to $\leq_2$ ([6]), thus we prefer to use the linear order to definition of uncertain OWA.

4 Structure of dataset. The practical problem of the fall detection

The aim of this study was the adaptation of a new approach for generating a set of rules in inference systems for data based on interval-valued fuzzy set theory and their application to fall detection systems (posture detection).

Data were collected by an inertial motion sensor and Kinect cameras that were assigned at the center of the ceiling or in front of the room. Because data in video format would violate privacy, only depth maps were used. Data received by the inertial motion sensor were sent using Bluetooth protocol, while depth maps were retrieved by USB protocol. Data collected from IMU sensors were prepared with the usage of source code contributed by the manufacturer, while depth map data were collected with the OpenNI library. Two Microsoft Kinect cameras provided 5990 depth images, which were collected and stored in the UR Fall Detection Dataset (accessible on [1]). Mentioned cameras were placed at different angles and registered 30 distinctive falls. Every fall is made up of approximately 150 frames that were saved in PNG16 format with the 640x480 dimensions. In this paper, we concentrate on studying a single depth map to detect a lying pose, even though the UR Fall Detection Dataset provides also character movement analysis. Parameters describing character posture were specified as a consequence of clustering 600 images showing form in different situations: depicting daily life activity, during fall, and laying. The following features characterize our problem:

- H/W - a ratio of width to height of characters box frame
- H/H_{max} - a ratio of the height of the character surrounding the box to the character's physical height
- $max(\sigma_x, \sigma_z)$ - the maximum chosen between the standard deviation of points included to the person form, from its center of gravity along X and Z axes, based on camera coordinate system
- $P40$ - some points included to the person lying in a cuboid 40 cm high, located above the floor, divided by a number of all points belonging to the person.

All of the above parameters were parsed to interval values after fuzzification, using the rule:
If : $fuzzy\_value$ **is** $None$ **then**
$fuzzy\_value = Method\ to\ imputation\ fuzzy\ interval\ values$
else : $fuzzy\_value = [fuzzy\_value, fuzzy\_value]$.

Fuzzy values/labels (*low*, *medium*, *high*) are built by using the function presented in [17]. For the case of missing data, we use the method (Algorithm 1) presented in [16] and based on similarity and knowledge measure. However, it is not the problem of lack of data that is being addressed in this paper.

5 Approximate inference system

We realize the problem of posture detection classification in the following points included in Figure 2 and applied interval-valued multi-conditional approximate reasoning.

The general schema of interval-valued multi-conditional reasoning has a form:

$$R_1: \ If\ x\ is\ \mathcal{D}_1\ then\ y\ is\ \mathcal{E}_1$$

$$R_2: \ If\ x\ is\ \mathcal{D}_2\ then\ y\ is\ \mathcal{E}_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$R_n: \ If \ x \ is \ \mathcal{D}_n \ then \ y \ is \ \mathcal{E}_n$$

$$fact: \ x \ is \ \mathcal{D}'$$

– –

$$y \ is \ \mathcal{E}',$$

where $\mathcal{D}_1, \cdots, \mathcal{D}_n, \mathcal{D}' \in IVFS(X)$, $\mathcal{E}_1, \cdots, \mathcal{E}_n, \mathcal{B}' \in IVFS(Y)$.

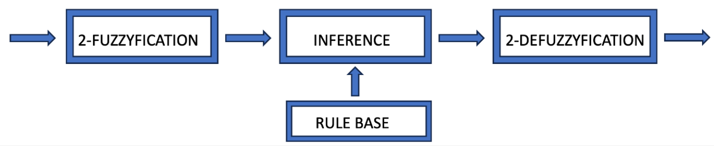

Fig. 2. Schema Approximate Reasoning

To determine $\mathcal{E}'$ we used the following method:

1. For each rule, the associated interval-valued fuzzy relation R_i is built, where $R_i \in IVFR(X \times Y)$ and $R_i(x, y) = \mathcal{A}_2(N_{IV}(\mathcal{D}_i(x)), \mathcal{E}_i(y))$ for $\mathcal{A}_2$ is interval-valued fuzzy aggregation function and N_{IV} is interval-valued negation;
2. The interval-valued aggregation functions $\mathcal{A}$, $\mathcal{A}_1$ and $\mathcal{A}_3$ are taken;
3. For each rule is calculated (GMP):
$$\mathcal{E}_i'(y) = \mathcal{A}_{x \in X}(\mathcal{A}_1(\mathcal{D}'(x), R_i(x, y))), \ with \ i = 1, \cdots, n;$$
4. Compute: $\mathcal{E}' = \mathcal{A}_{3i=1,\ldots,n}(\mathcal{E}_i')$.

We generalized the fuzzy inference, so in the process of aggregating premises, a generalization of the proposed ([17]) method consists of the combination of aggregation and measure of knowledge as the following new operator:

$$O_F = \mathcal{B}(\mathcal{A}_{i=1}^{n}(x_i), K(F)), \tag{2}$$

where F is an interval-valued fuzzy set, i.e. premises data in a given rule and K knowledge measure proposed by us in [16] and $\mathcal{A}, \mathcal{B}$ are interval-valued aggregation functions.

The weighted average method was used in the defuzzyfication block, and a threshold value of 0.5 was adopted in the step of arriving at the final decision.

5.1 The Expert's opinion used to create rules

In order to label the pose of the character, the following labels were used: *isLy* for a lying, *notLy* for not a fall, and *mayLy* for a state where a person is about to fall. Relying on gathered data a rule set was created. As a result of the expert selection of parameters for three-value labeling (low, medium, high) of each of the values of the four attributes, 3^4 was obtained, i.e. 81 rules, (see [14]): 16 rules for isLy, 13 for notLy, and 52 for mayLy pose.

For example, we have the following rules:

– R2: if P40 is low AND HW is high AND Sigma is low AND HHmax is medium THEN Pose is not_Ly,
– R6: if P40 is low AND HW is high AND Sigma is medium AND HHmax is low THEN Pose is may_Ly,
– R78: if P40 is high AND HW is low AND Sigma is medium AND HHmax is low THEN Pose is is_Ly.

5.2 Machine learning used to create rules

In many articles, Artificial Intelligence techniques have been applied to the same goal [11], [22], [19] In this article, the decision tree has been obtained from the database using the Scikit-Learn library, and then it was used to create a new set of rules. The portion of the decision tree obtained is shown in Fig. 3.

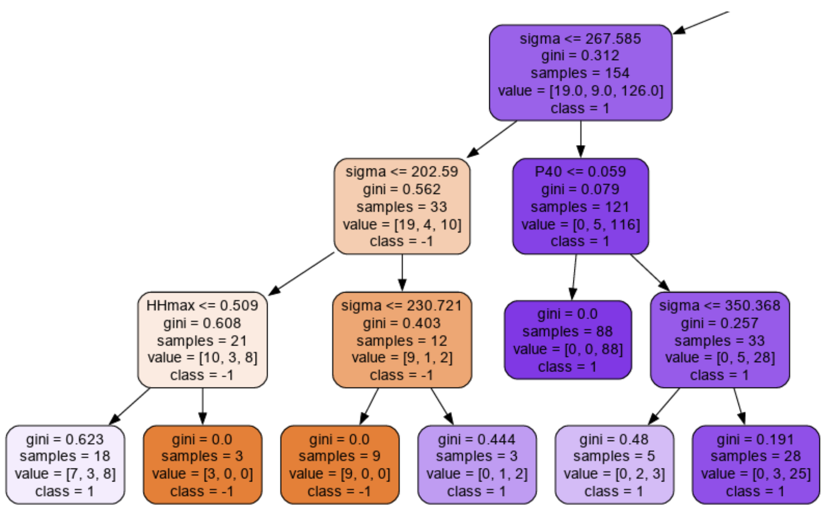

Fig. 3. Piece of generated tree

The parameters of the tree were set as follows:

– split criterion: Gini,
– maximal depth: 6,
– minimal number of samples to create a leaf: 4,
– minimal number of samples to split: 9,
– splitter: random.

From the tree, we obtained 16 rules, and 3 of them are shown below:

– R1: if HW is low AND Sigma is low AND HHmax is medium THEN Pose is not_Ly,

- R2: if Sigma is high AND HHmax is low THEN Pose is may_Ly,
- R8: if P40 is high AND HHmax is low THEN Pose is is_Ly.

All are included in Github https://github.com/PGrochowalski/ifis, where we may also find a description of the library for inference systems with uncertainty in Python.

5.3 Proposed new hybrid methodology

The main difference and novelty between the proposed approach and others known in the literature concern the method of assessing the quality of machine learning rules and combining them at a given efficiency with expert rules.

Procedure based on a new approach rules

The proposed method is based on the support of rules generated with the help of experts, i.e. expanding the set of these rules with rules obtained using the machine learning method, i.e., decision trees. The rules obtained by the above-mentioned method are assessed using the measure of decision-making effectiveness [21]:

$$EFF(R_i^{ML}) = \frac{card(Support_{C_l}(R_i^{ML}))}{card(Support_U(R_i^{ML}))}.$$

The quality of EFF rules is therefore determined based on the quotient of the number of objects correctly classified by a given rule for a given class by the number of all objects correctly classified by the analyzed rule. Rules from decision trees that achieved a given level in the EFF measure in the context of the effectiveness of expert rules were fed into their set, obtaining the so-called hybrid set of rules (Step 2 in Algorithm 1). However, an important element of the presented algorithm was Step 1, in which we evaluate both sets of expert rules R^E and machine learning R^{ML} in terms of logic and optimization, i.e. we eliminate rules that are contradictory to both sets, and we also minimize /optimize the description of conditions for individual attributes.

After evaluating the decision tree we got 16 rules. The next step was to check which of them were contradictory to the rules presented by the expert, and 3 of them were. Therefore they were excluded from the set. All of the rules from the decision tree that had EFF value less than 0.9, also were excluded, and that was only 1 rule. Afterward, we took a closer look at premises in generated rules. If we found a generated rule, that is similar (has the same result and premises that occur are the same) to the expert rule, but the generated one has fewer premises, then the expert rule was excluded from the set. In this procedure, 21 rules from the expert rule set were excluded. The mixed rule base was created out of 60 rules withdrawn from the expert rule base, and 11 generated one.

Algorithm 1: Mixed rules construction algorithm

Notations:
$U = \{C_l\}_{l=1,...t}$ - set of t decision class;
$R^{ML} = \{R_i^{ML}\}_{i=1,...,n}$ - set n rules obtained by machine learning method;
$R^E = \{R_i^E\}_{i=1,...,m}$ - set m rules obtained by experts' knowledge;
R^{MIX} - set of rules created from the R^E and R^{ML} set - mixed rules;
$A = \{A_k^{R_i^{ML}}\}_{k=1,...,z}$ - $k-th$ premise in the form as condition for $i-th$ rule from R^{ML}.
Input : R^E and R^{ML};
Output: R^{MIX};
begin
Step 1. Evaluation rules - Logical analysis of rules and optimization of premises.
1. Elimination of conflicting rules from R^{ML} to R^E;
for *i=1,...,n* **do**
  for *j=1,..,m* **do**
    if $R_i^{ML} : A \rightarrow b \wedge R_j^E : A \rightarrow c$ **then**
    $\backslash *$ opposite conclusions for consistent premises
    $R^{ML} \leftarrow R^{ML} \backslash R_i^{ML}$
2. Optimization of premises/rules;
2.1. **if** $\bigcap_{t=1,...m} A_{k_t}^{R_i^{ML}} \neq \emptyset$ **then**
$\backslash *$ m - the number of conditions defining one of the features.
$A_k \leftarrow A_{k_1} \cap ....;$
2.2. **if** $A_k^{R_j^E} \cap A_l^{R_i^{ML}} \neq \emptyset$ *and* $card(k) \leq card(t)$ **then**
$R^{ML} \leftarrow R^{ML} \backslash R_i^{ML}$; **else** $R^E \leftarrow R^E \backslash R_j^E$;
return R^E *and* R^{ML};
Step 2. Creation of R^{MIX}
$R^{MIX} := R^E$;
for *l=1,...,t* **do**
  for *i=1,...,s* **do**
    $\backslash *$ s - number rules in R^{ML} after Step 1.
    if $EFF(R_i^{ML}) = \frac{card(Support_{C_l}(R_i^{ML}))}{card(Support_U(R_i^{ML}))} \geq \min_{k=1,...p} EFF(R_k^E)$ **then**
    $\backslash *$ p - number rules in R^E after Step 1 and *Support* means set of objects supporting i-th rule.
    $R^E \leftarrow R^E \vee R_i^{ML}$;
    $R^{MIX} := R^E$;
return R^{MIX}

6 Experimental results and discussion

We compare the impact of the proposed new hybrid method for building rules to expert rules and using machine learning to create rules.

We analyze the three sets of models:

Model 1 Inference system based on experts rules;
Model 2 Inference system based on machines' learning rules;
Model 3 Inference system based on a hybrid method for creation rules.

Model 1 is characterized by experts' rules (see 5.1.) and studied in [16].
In the case of model 2, we have rules obtained using one of the machine learning methods, i.e. decision trees.
However, we call model 3 hybrid because, using Algorithm 1, we obtained the set of expert rules by expanding the set of rules from the decision tree with a purified set of rules with a high-efficiency index.

Each model was rated and their effectiveness was determined by the following characteristics ACC, SENS, SPEC, PREC, and the obtained results are presented in Table 1:

- accuracy $ACC = \frac{TP+TN}{TP+TN+FP+FN}$,
- specificity $SPEC = \frac{TN}{TN+FP}$,
- precision $PREC = \frac{TP}{TP+FP}$,
- sensitivity $SENS = \frac{TP}{TP+FN}$,
  where TP is the number of correct isLy classifications, TN is the number of correct notLy classifications, FP is the number of notLy classifications as isLy, and FN is the number of isLy classifications as notLy.

Table 1. Performance of models

Models	ACC	SENS	SPEC	PREC
Model 1	0,969	0,962	0,999	0,875
Model 2	0,977	0,995	0,922	0,975
Model 3	**0,993**	**0,998**	0,976	**0,992**

Using the mean square and minimum, respectively, in formula (2), we obtained results that presented the effectiveness of our three research models in Table 1. From Table 1, we can deduce a clear improvement in the efficiency of the classification based on the hybrid, i.e., mixed rules. The increase in both error observation measures for our proposed hybrid rule fusion method (Model 3), i.e. accuracy and precision, to the level of 99.3% and 99.2%, respectively, proves the very good degree to which we classify correctly. Moreover, a very high Sensitivity score of 99.8% means that a "bad" diagnosis carries a small chance of error, which is important in a social/psychological assessment.

7 Summary and future works

In this paper, we focused on the use of a new technique to create a set of rules, with uncertainty representation using interval-valued fuzzy set theory in approximate inference in posture detection, a fall detection system for the elderly. The set of rules and their genesis play a key role, regardless of the source: expert, or created using machine learning, also carries uncertainty as to the source of data, measurements, or human assessments. Therefore, in this paper, we studied an approach that takes into account both sources/methods of creating a set of rules used in the generalized approximate inference model, i.e. a hybrid method of generating a set of rules in the interval-fuzzy inference model. The practical aspect, i.e. detecting falls in older people, is an important issue and necessary for support in modern, highly developed societies, therefore the very high effectiveness of posture detection achieved will ensure an increase in the safety of many elderly people. In the future, we will apply the issue of hybrid rule creation techniques to situations with data privacy problems, with an indication of the use of federated learning.

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